Graph Realization in IoT: Theory, Practice and New Trends

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Outline

1、 Introduction

2、 Related Theory

3、 Practice

4、 New Trends

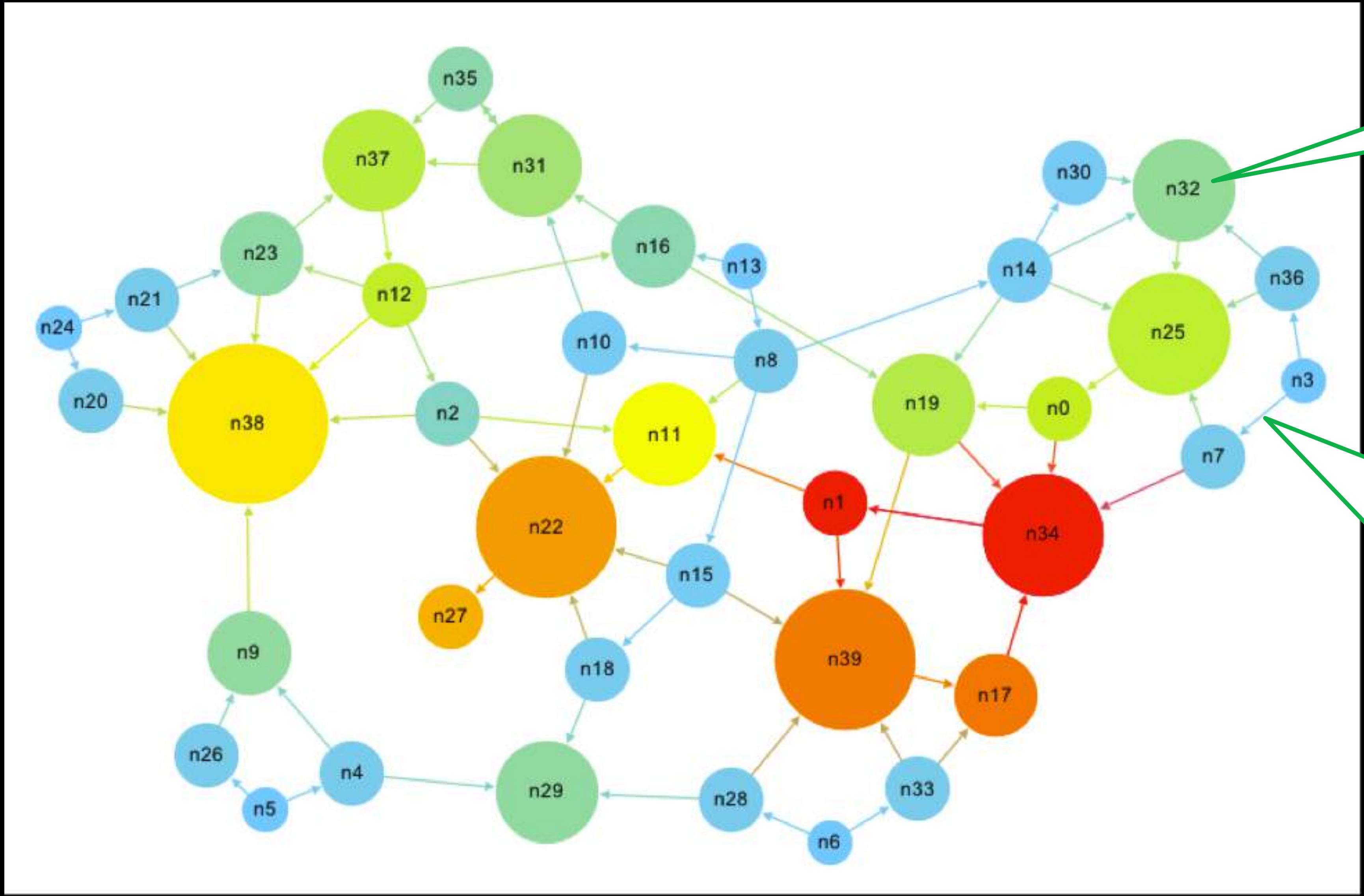
Internet of Things (IOT) Will Cover Our Life Space

Sensors and Actuators collect information, analyze real-time contexts, and provide smart services.

Things are connected by invisible links.

What is the ***internal structure*** of this complex network?

IOT can be modeled by a Graph

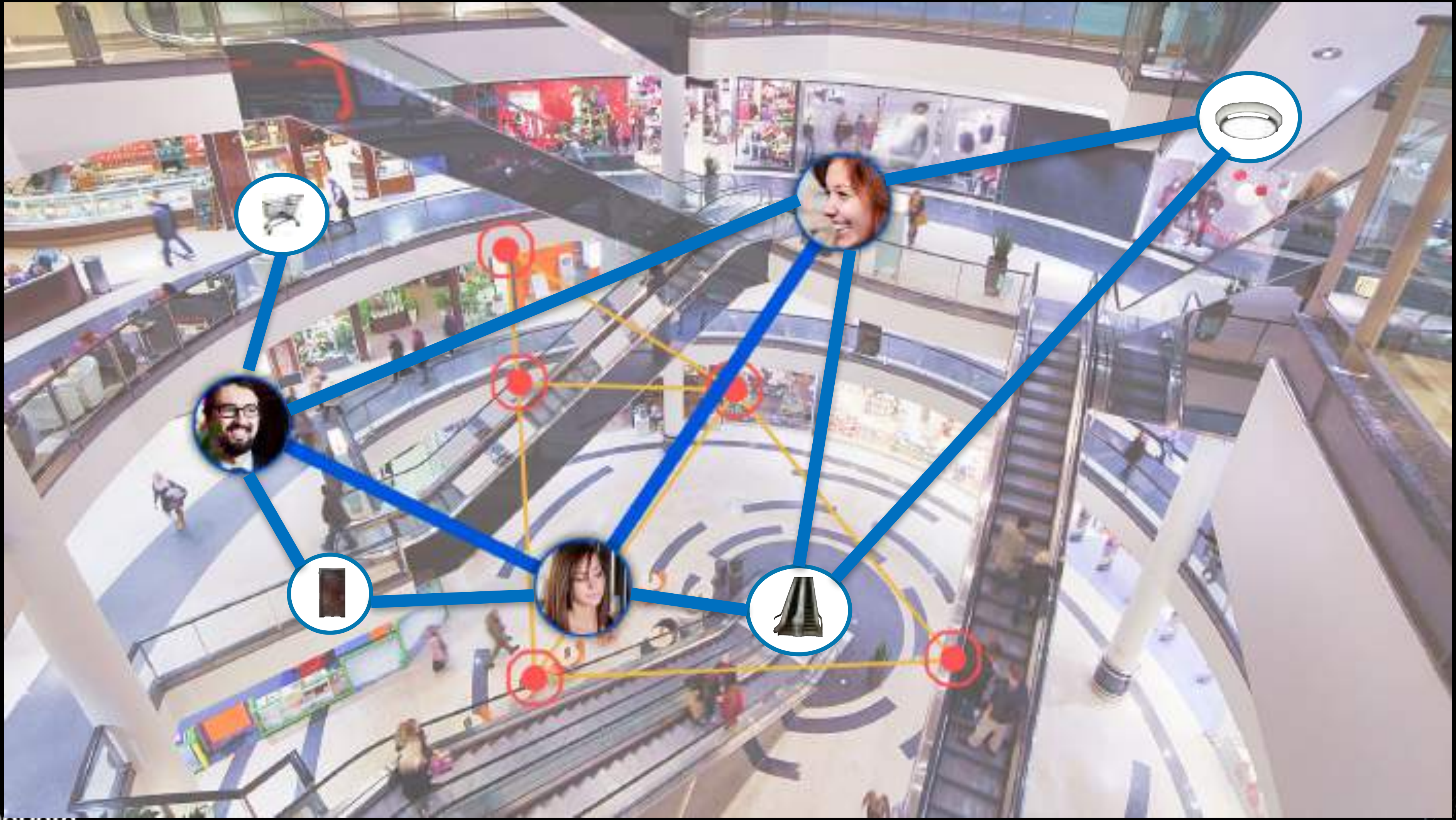


People or entity,
as Nodes

Links
measure
relationships:
distance,
similarity,
friendship
etc.

Realize the Graph helps to reveal the internal structure of IOT

An Example: Localization in IOT

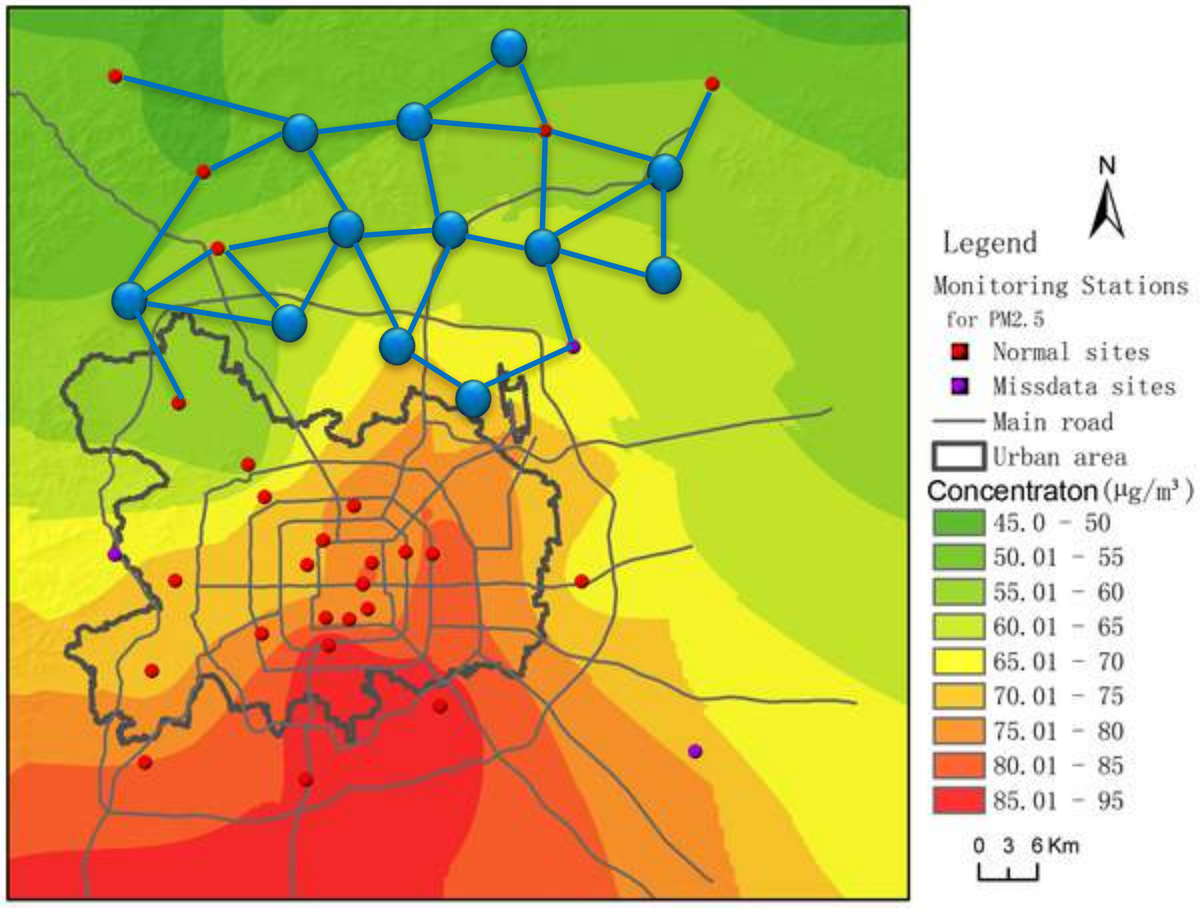


What is the real-time geometric structure of the network?

Locations of entities can be inferred by distance links among entities.

Realize the Graph helps to reveal the internal structure of IOT

Another Example: Data Inference in IOT



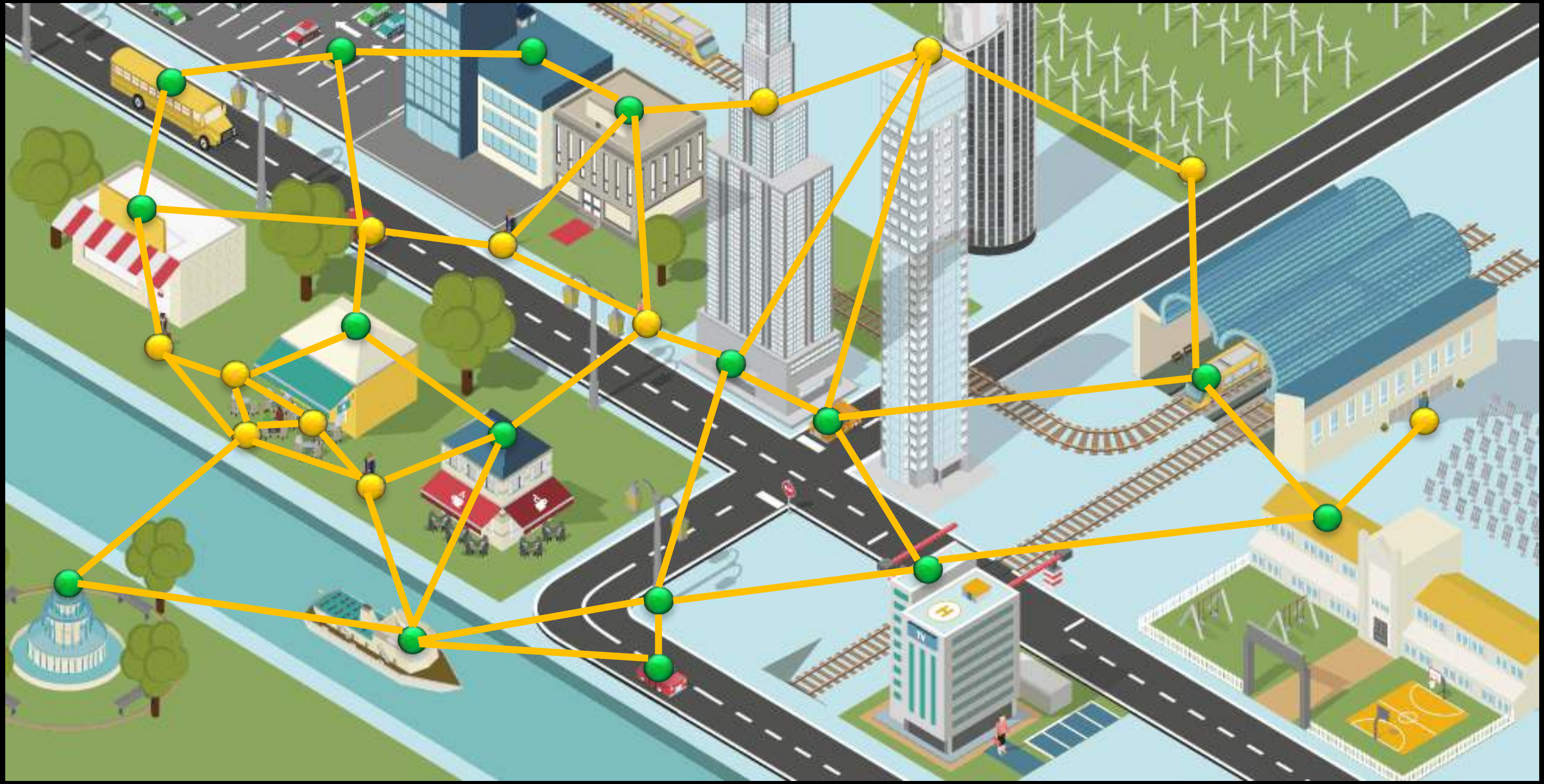
Having only limited number of monitoring stations.

We want to know exact PM2.5, AQI data of more spots.

The data can be inferenced by similarity (distance) links among the spots and the monitoring stations.

Realize the Graph helps to reveal the internal structure of IOT

Another Example: Network Visualization in Smart City



To visualize all entities onto the city map.

Locations of people and entities can be inferred by the links among the non-GPS entities, and entities with GPS.

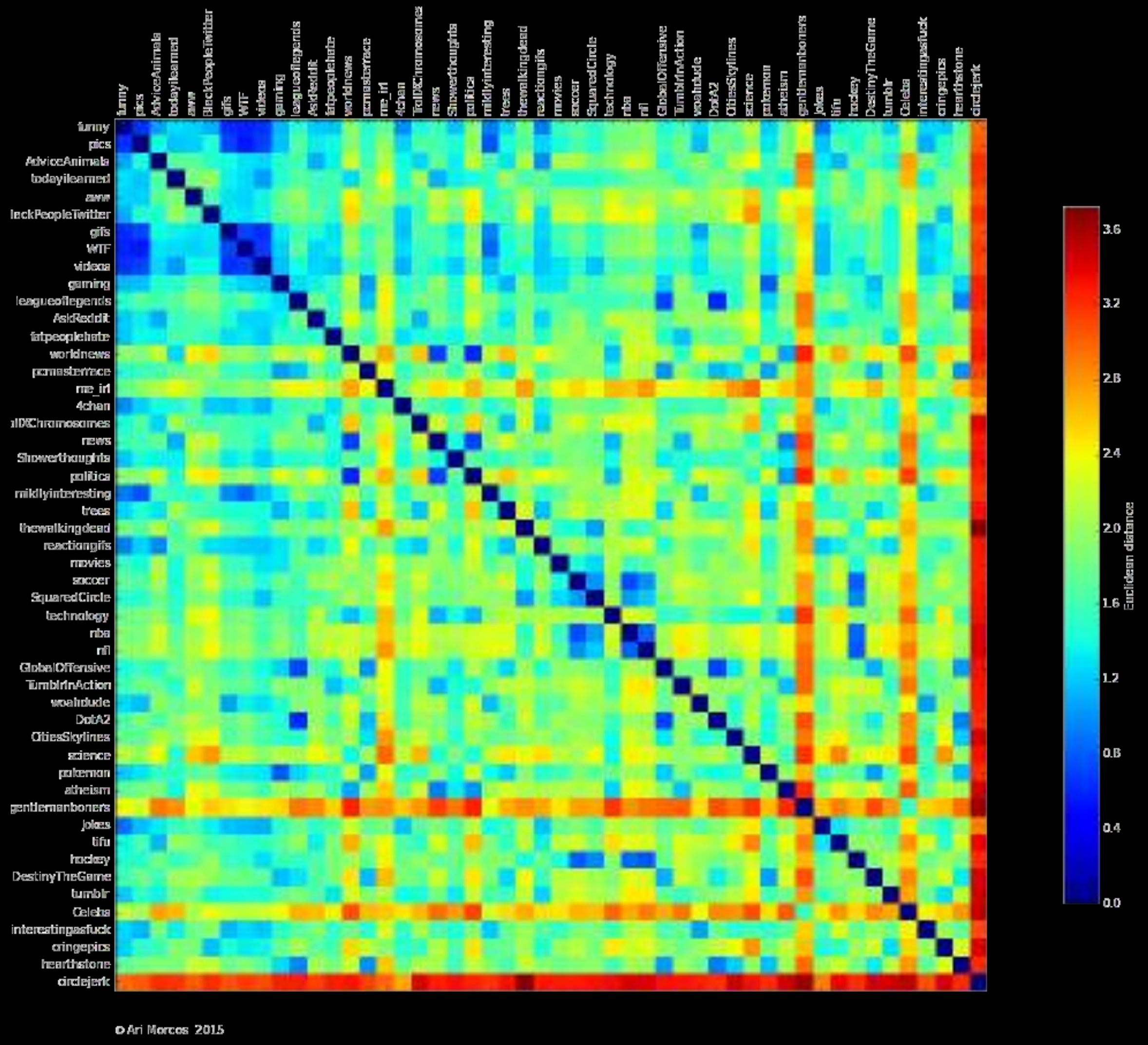
Graph Realization Problem

Definition 1: Graph Realization

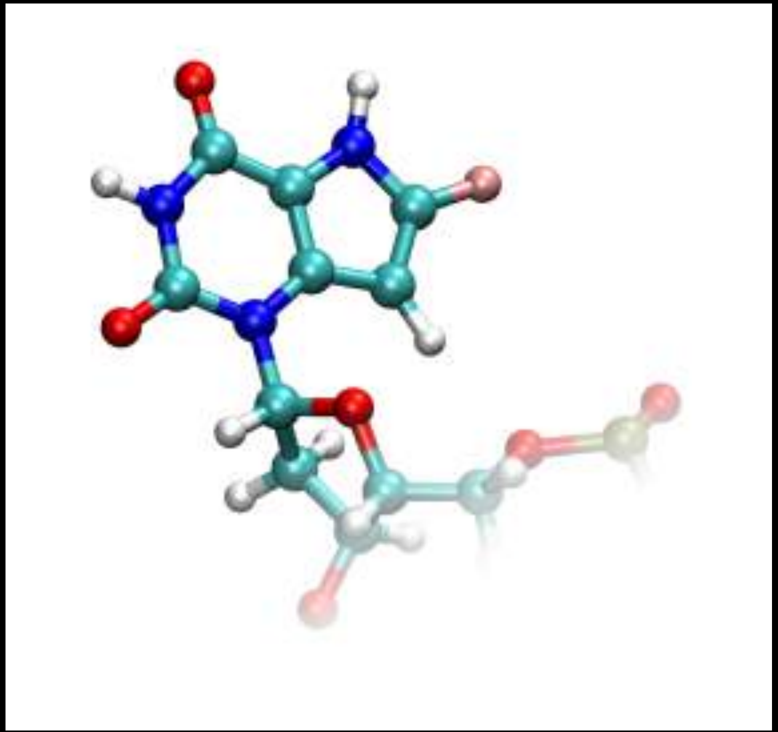
"Given a graph $G=(V, E)$ of $|V|=n$ vertexes and $|E|= m$ edges, along with a real number $\omega_{i,j}$ associated with each edge (i, j), ***graph realization*** is to find vertex coordinates $P=\{p_1, p_2,\ldots, p_n\}\in \Re^d$, such that the Euclidian distance between any two vertexes i, j equals to the number $\omega_{i,j}$."

To find the vertex coordinates in *d*-dimensional space by partially measured distances or similarities among some pairs of vertexes.

Graph Realization Problem



Given this distance matrix of DNAs



Realized DNA Structure

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Hardness of Graph Realization

Given $G = (V, E, \omega)$ and integer k, the problem: Does G have a realization in $\Re^k$, is NP-hard. [Saxe1979].

Is it really that difficult 

Saxe proves the NP-hardness using very special graphs, which needs special combinations of edge lengths, implying specific algebraic relations among the vertex. Such cases are very special and rarely appeared.

Generic Graph Realization

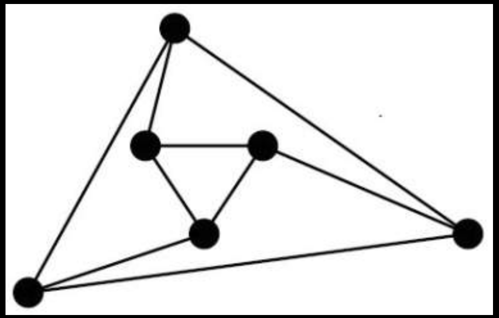
A realization is generic if the vertex coordinates are algebraically independent over the rationales.

The set of generic realizations is dense in the space of all realizations, and almost all realizations are generic.

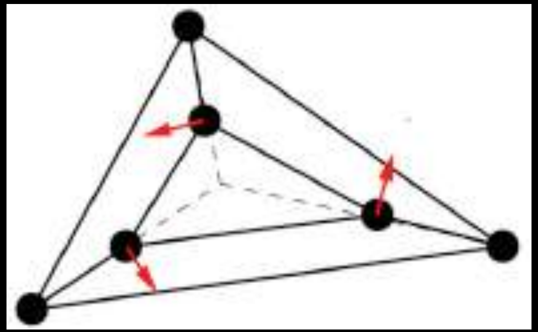
In generic graph, the graph realization problem can be approached from purely graph theory that ignores edge lengths. The generic graph realization problem is tractable.

Uniqueness of Generic Graph Realization

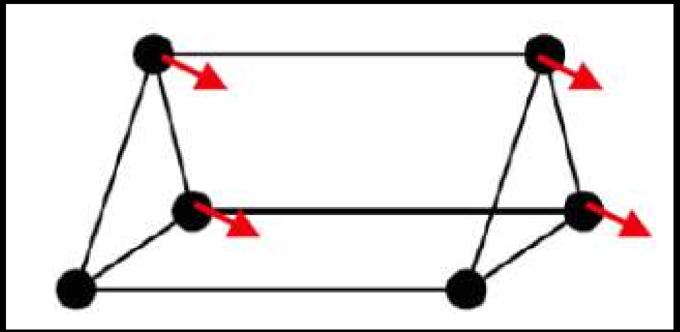
A generically rigid graph can have ***rigid, infinitesimally flexible, or even flexible*** frameworks.



rigid



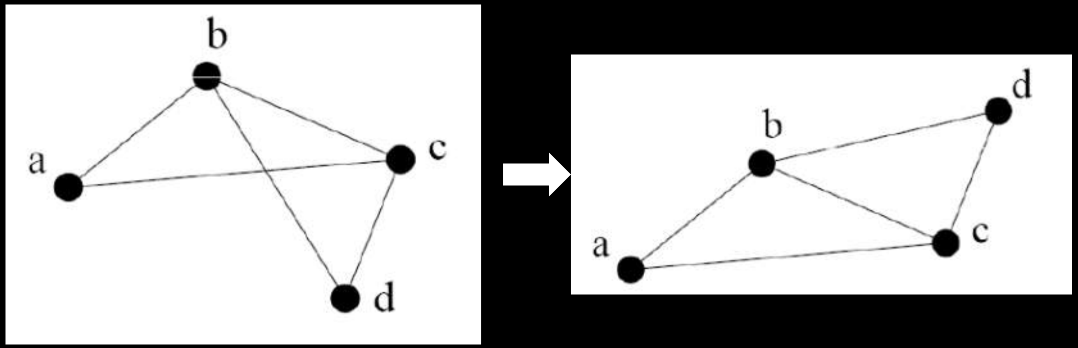
infinitesimally flexible



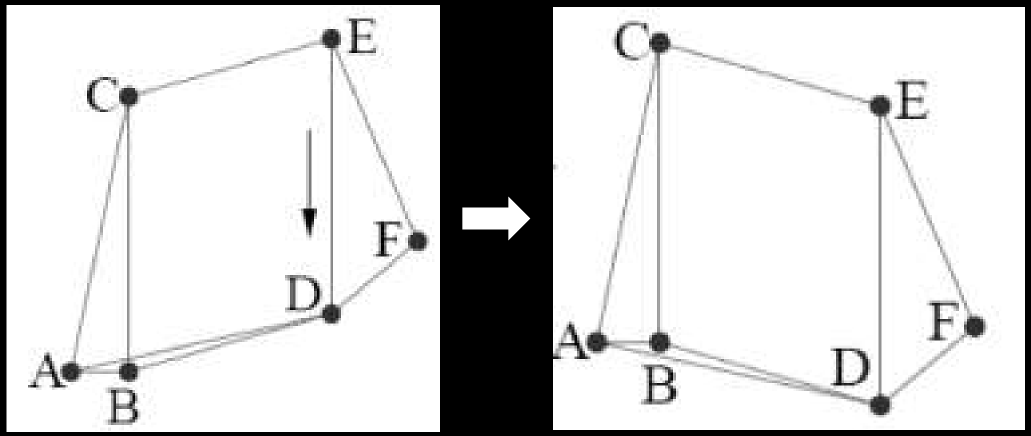
flexible

No unique realization solution

Rigid graph may also have multiple realizations

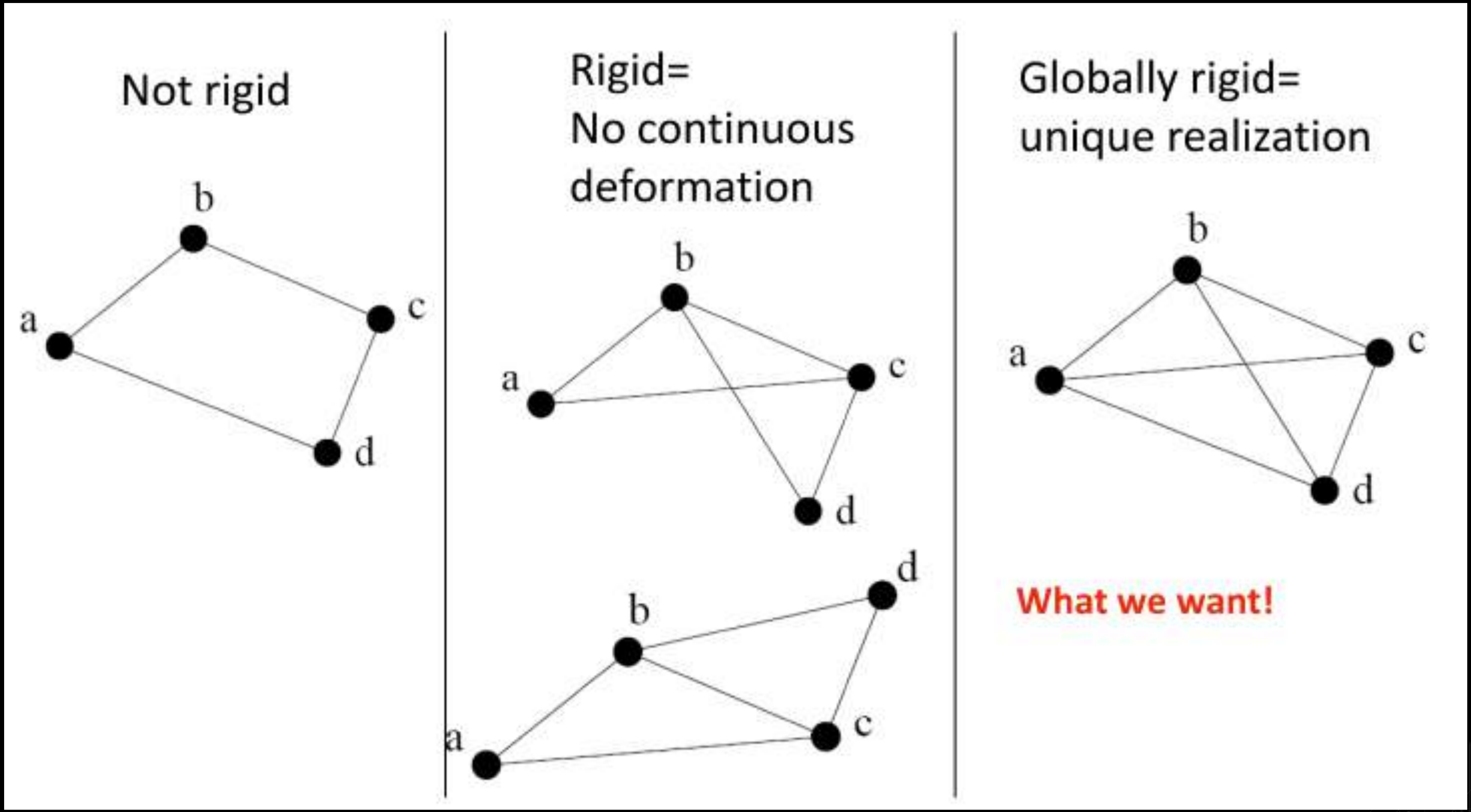


Flip ambiguity



Flex ambiguity

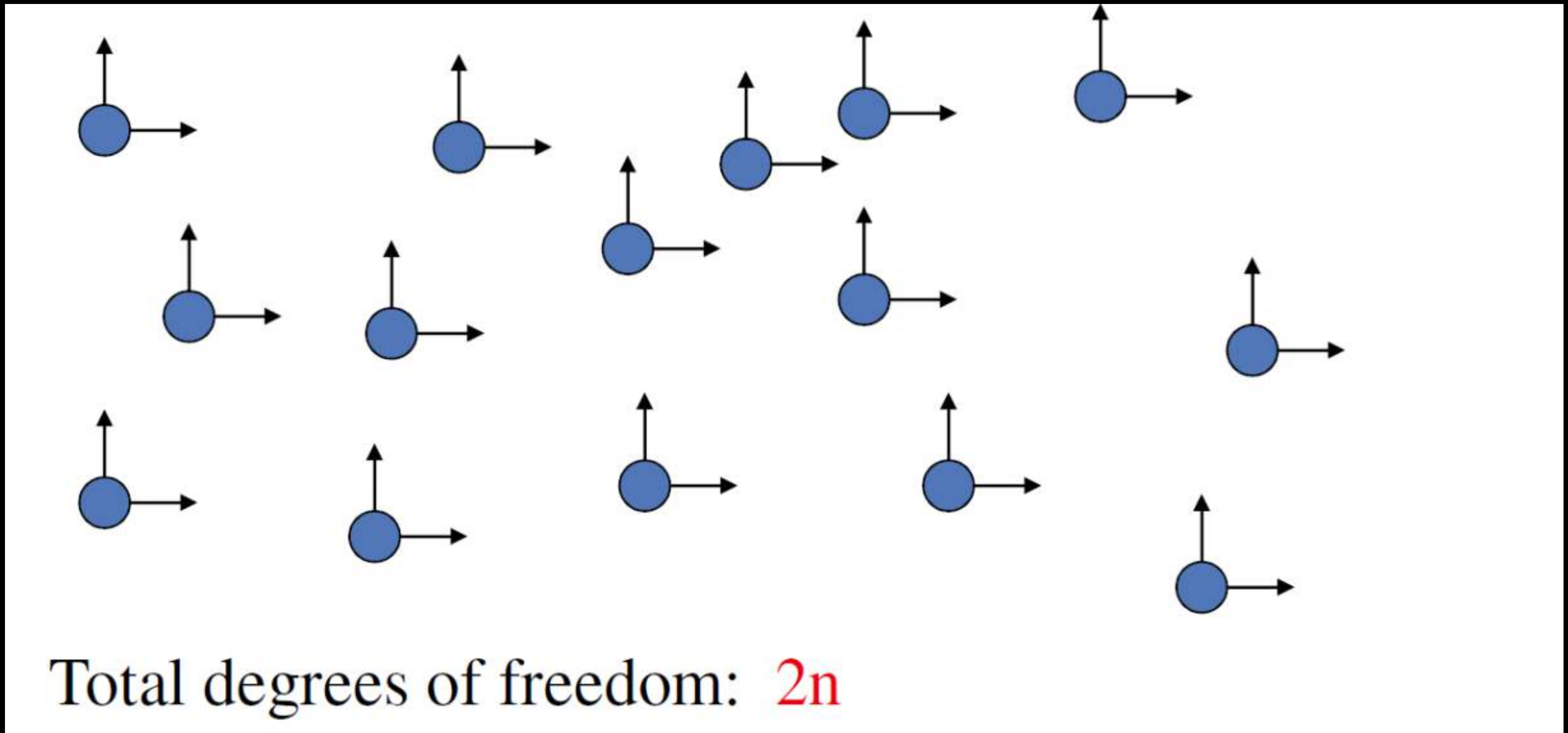
Global Rigidity: Condition of Being Unique Realization



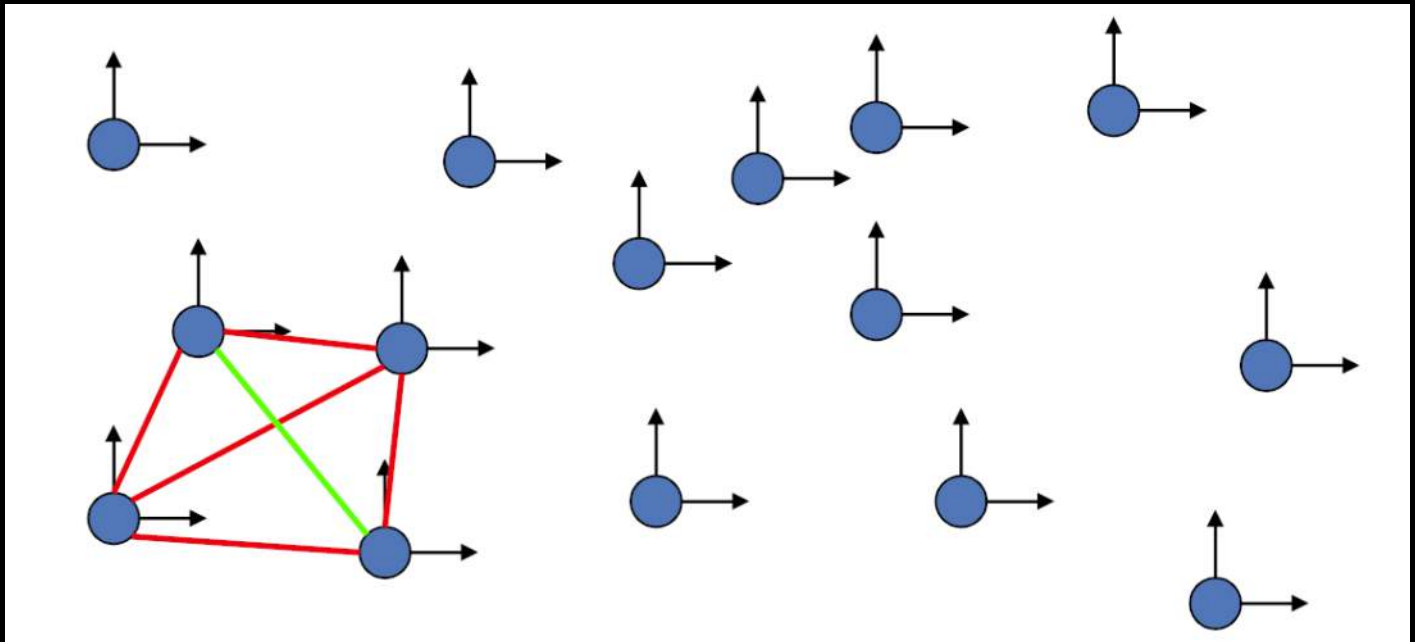
Intuition on rigidity (not global rigidity yet)

How many distance constraints are necessary to limit a framework to only trivial motion?

How many edges are necessary for a graph to be rigid?



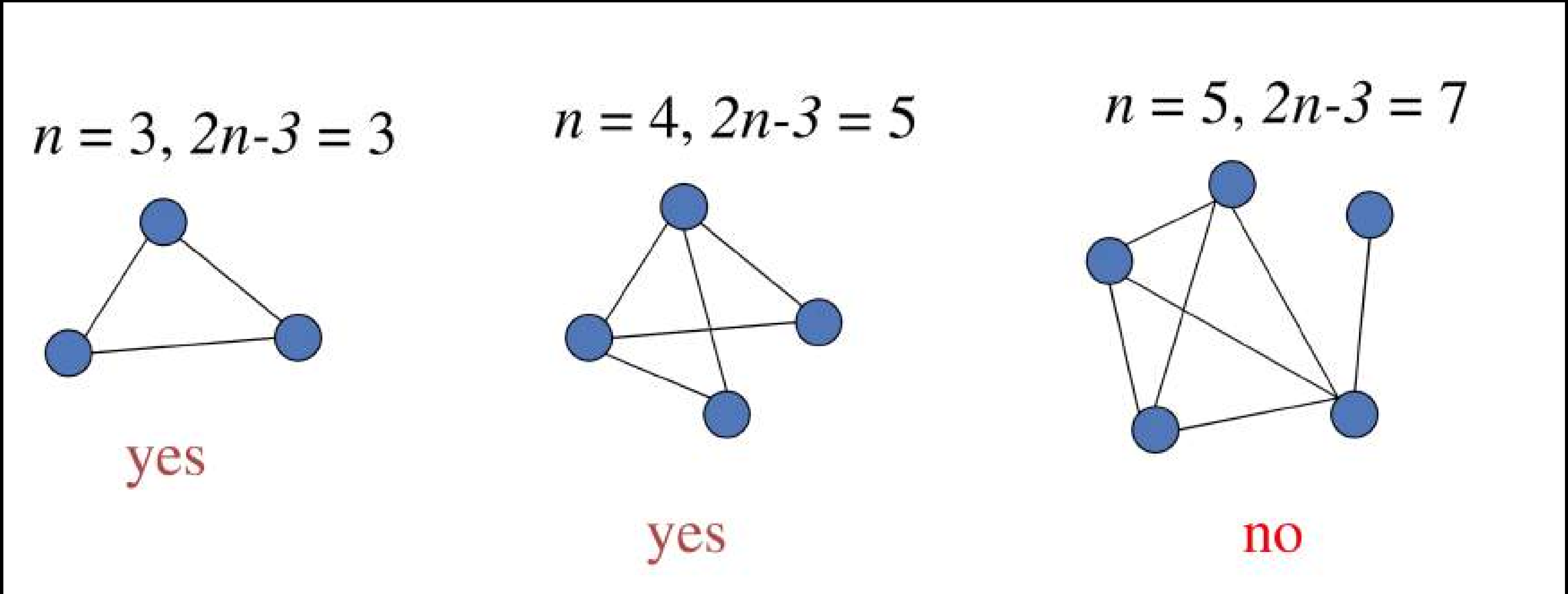
How many edges are necessary to make a rigid graph?



Each edge can remove a single degree of freedom

Rotations and translations will always be possible, so at least **2n-3** edges are necessary for a graph to be rigid.

Are 2*n*-3 edges sufficient?



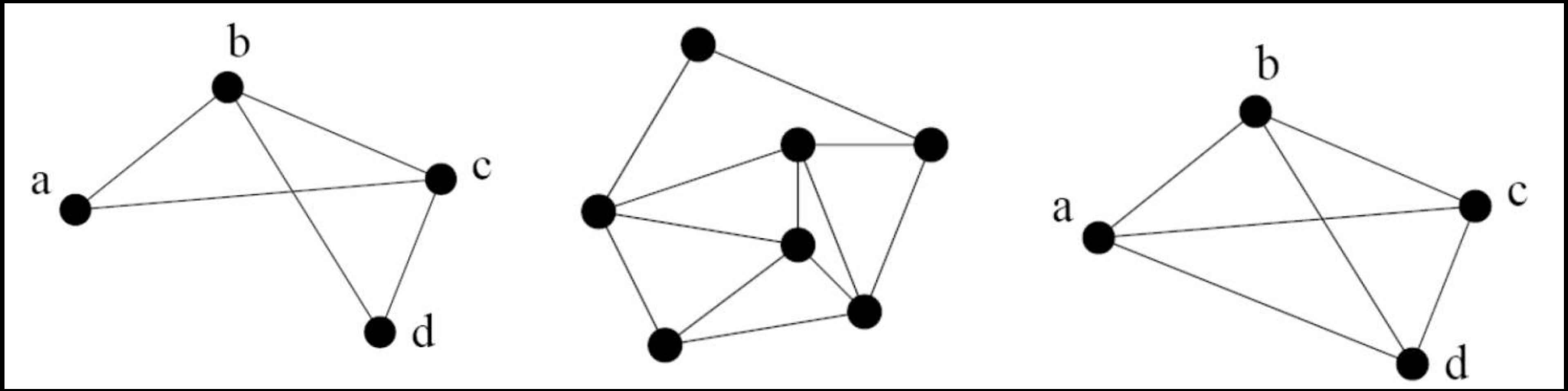
Further intuition

- Need at least 2n-3 “well-distributed” edges.
- If a subgraph has more edges than necessary, some edges are redundant.
- Non-redundant edges are independent, i.e., they remove a degree of freedom each.
- Therefore, 2*n*-3 independent edges guarantee rigidity.

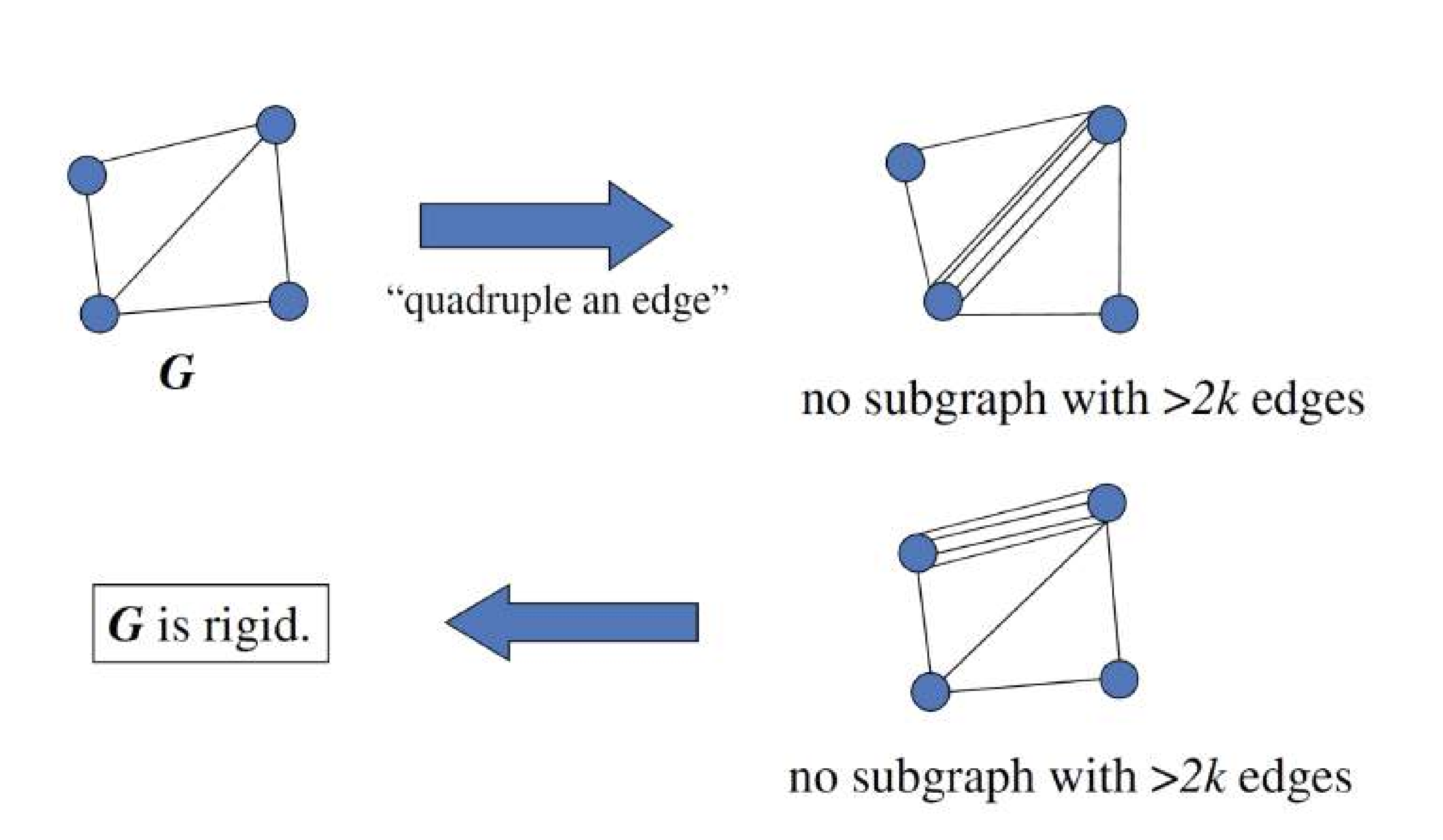
Laman condition

Laman graph: it has 2n-3 edges and no subgraph of k vertices has more than 2k-3 edges.

Laman condition: A graph is rigid if it contains a Laman graph.

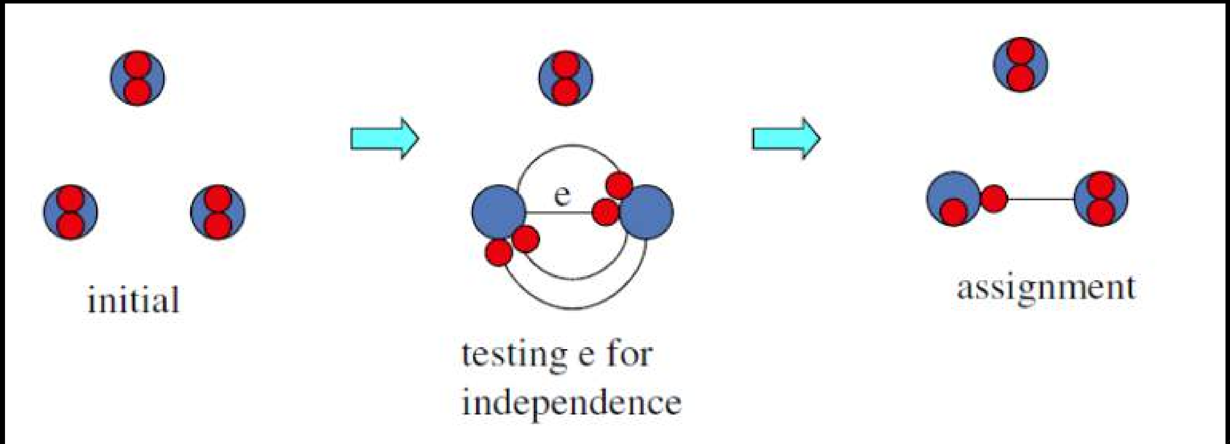


Algorithm to test rigidity

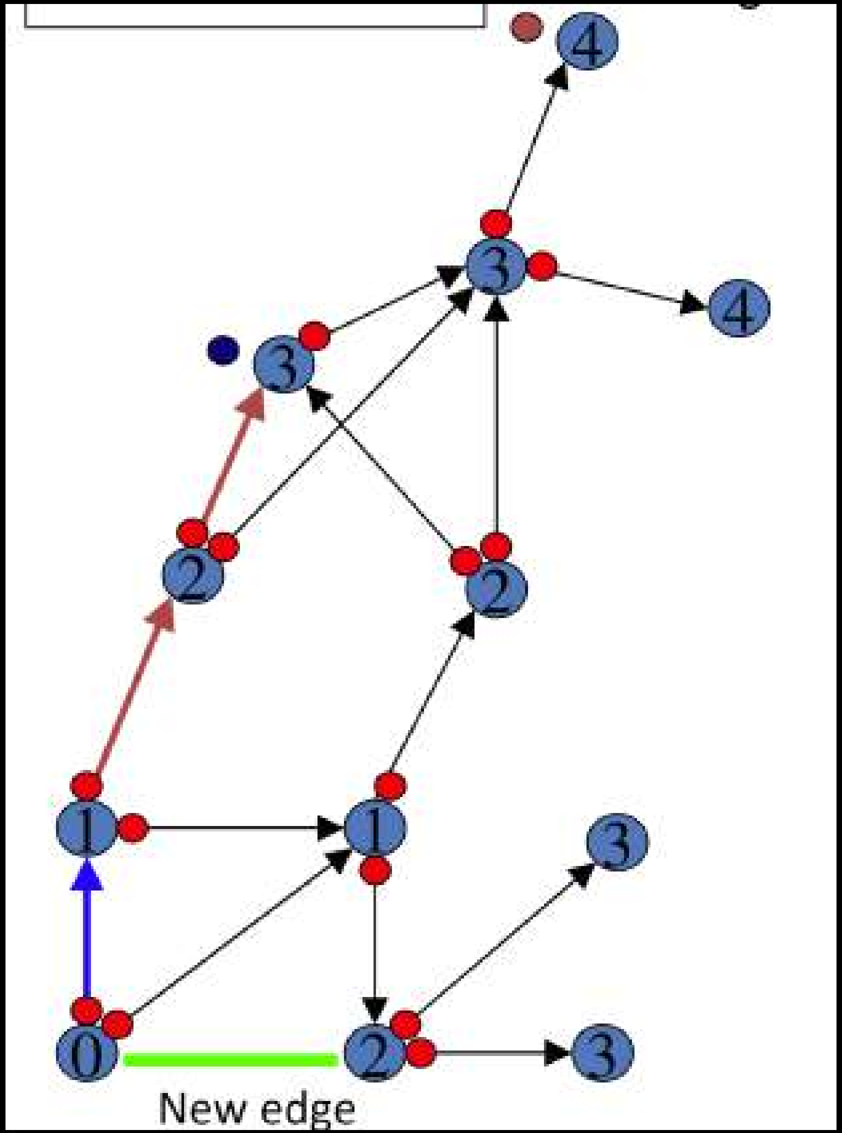


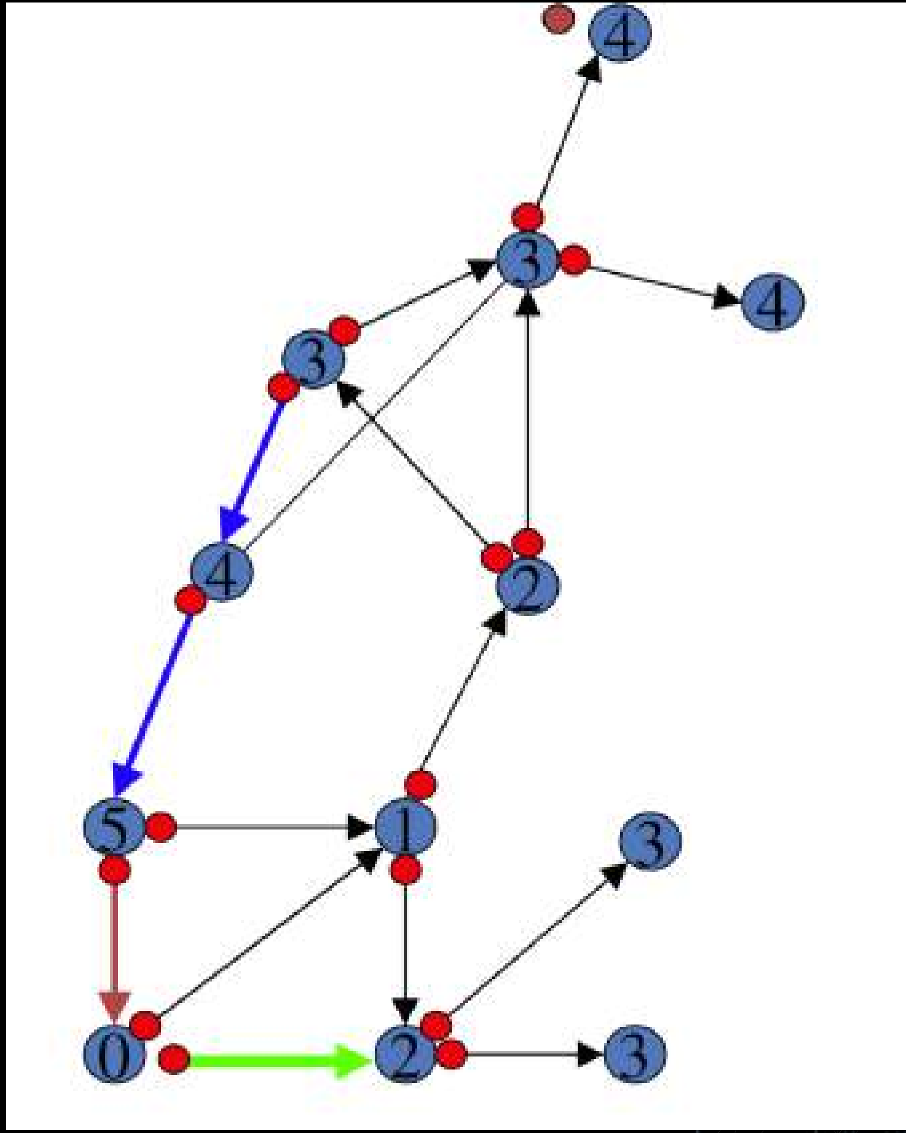
Algorithms to test Rigidity: Pebble Game

- Each node is assigned 2 pebbles. 2n pebbles in total.
- An edge is covered by having one pebble placed on either of its ends
- A pebble covering is an assignment of pebbles so that all edges in graph are covered.
- Test if a new edge is independent of the existing set: quadruple the edge; find a pebble covering for the 4 new edges.

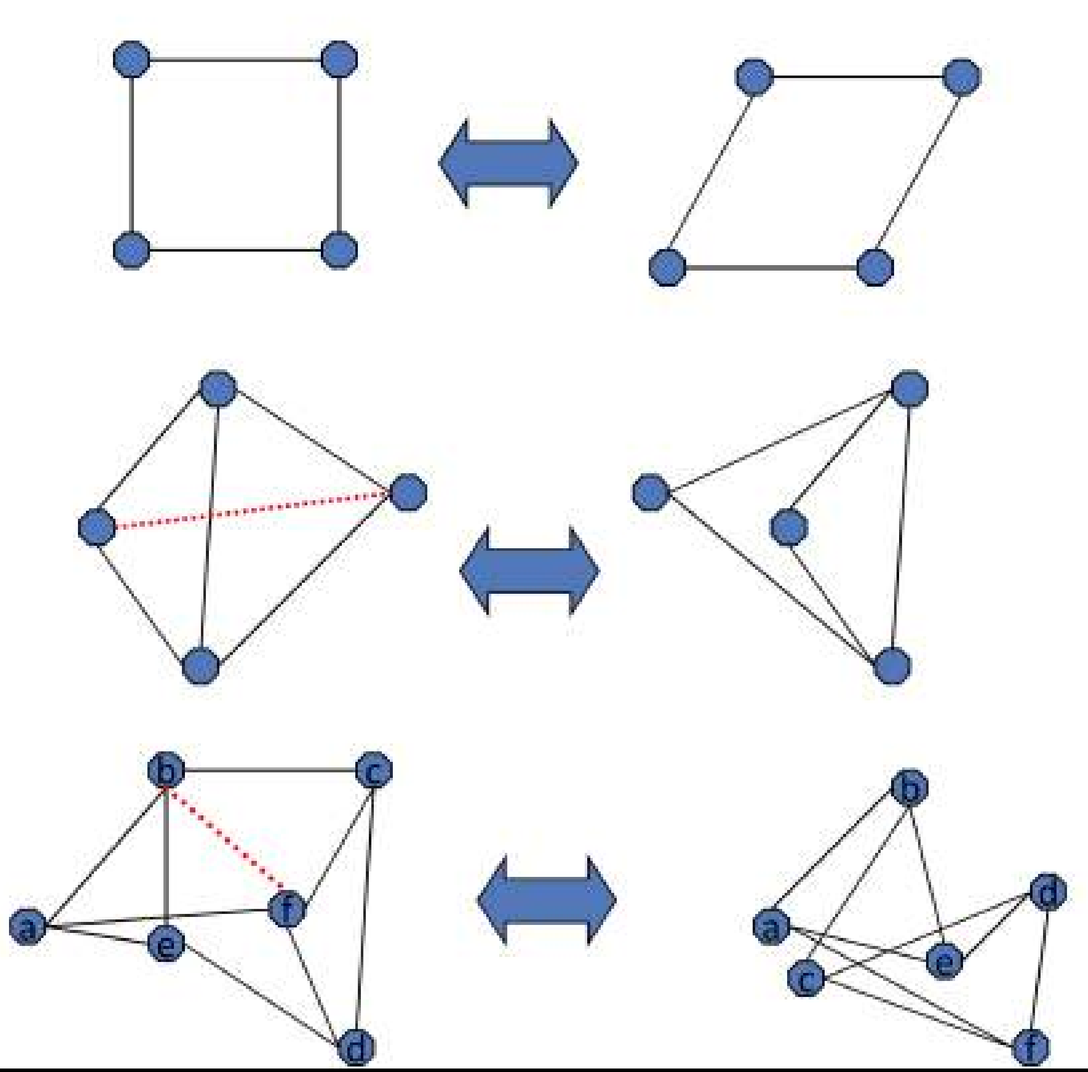


Pebble Game: Testing Newly Added Edge





Conditions of being Globally Rigid in 2-D



Solution:

G must *rigid*

G must be 3-connected, i.e. Connected after removal of 2 Vertices.

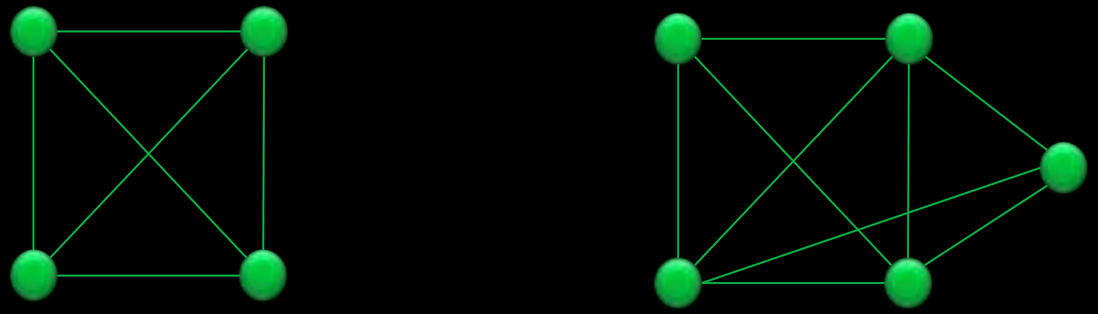
G must be *redundantly rigid:* It must remain rigid upon removal of any single edge

Conditions of being Globally Rigid in 1-D, 2-D

Let G be a globally rigid graph in $\Re^d$. Then either G is a complete graph on at most $d + 1$ vertices, or G is

(a) $(d + 1)$-connected; and

(b) redundantly rigid in $\Re^d$.



This is necessary and sufficient condition for only d=1, 2

Conditions of being Globally Rigid in *higher dimension*

For higher dimensions, the condition is only necessary :

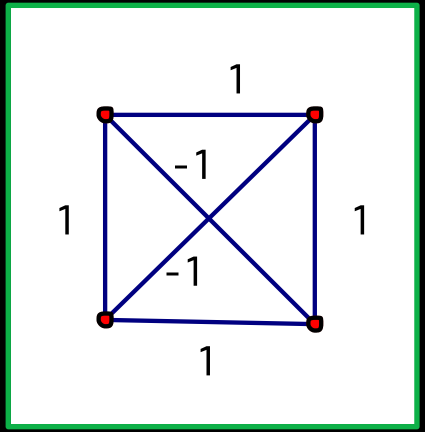
Let G be a globally rigid graph in $\Re^d$. Then either G is a complete graph on at most $d + 1$ vertices, or G is
(a) $(d + 1)$-connected; and
(b) redundantly rigid in $\Re^d$.

Necessary and Sufficient Conditions of being Globally Rigid in *higher dimensions*

Equilibrium stress :

For a framework G(**p**), an *equilibrium stress* ω is an assignment of a scalar $\omega_{ij} = \omega_{ji}$ to each pair of distinct vertices {i,j} of G, such that $\omega_{ij} = 0$ when {i,j} is not an edge of G, and for each i, the equilibrium equation $\Sigma_j\, \omega_{ij}(\mathbf{p}_j - \mathbf{p}_i) = 0$ holds.

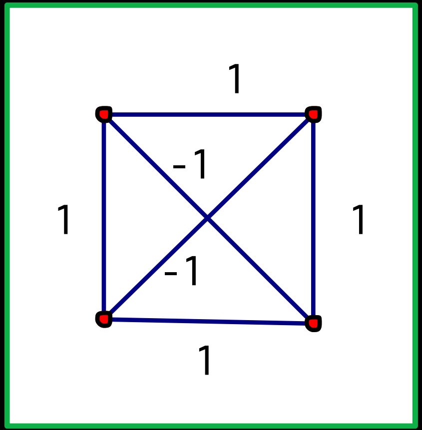
The following is a square with its equilibrium stresses indicated:



Necessary and Sufficient Conditions of being Globally Rigid in *higher dimensions*

Stress **matrix:**

Given an equilibrium stress ω for a framework G(**p**) with n vertices, the *stress matrix* Ω is the n-by-n symmetric matrix where the (i,j) entry is $-\omega_{ij}$ and the diagonal entries are such that all the row and column sums of Ω are 0.



$$\Omega = \begin{bmatrix} 1 & -1 & 1 & -1 \\ -1 & 1 & -1 & 1 \\ 1 & -1 & 1 & -1 \\ -1 & 1 & -1 & 1 \end{bmatrix}$$

Stress matrix

Necessary and Sufficient Conditions of being Globally Rigid in *higher dimensions*

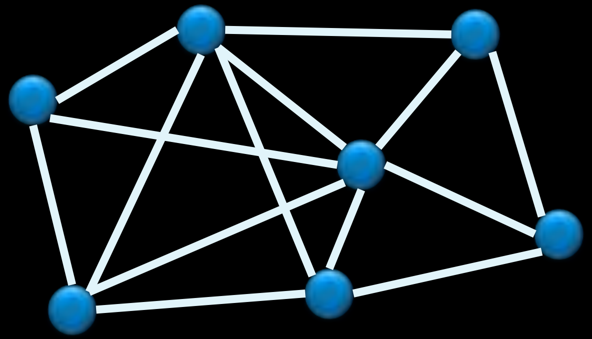
Let (G; p) be a generic framework in $\Re^d$ on at least d+2 vertices. (G; p) is globally rigid in $\Re^d$ if and only if (G; p) has an equilibrium stress ω for which the rank of the associated stress matrix Ω is |V| -d -1.

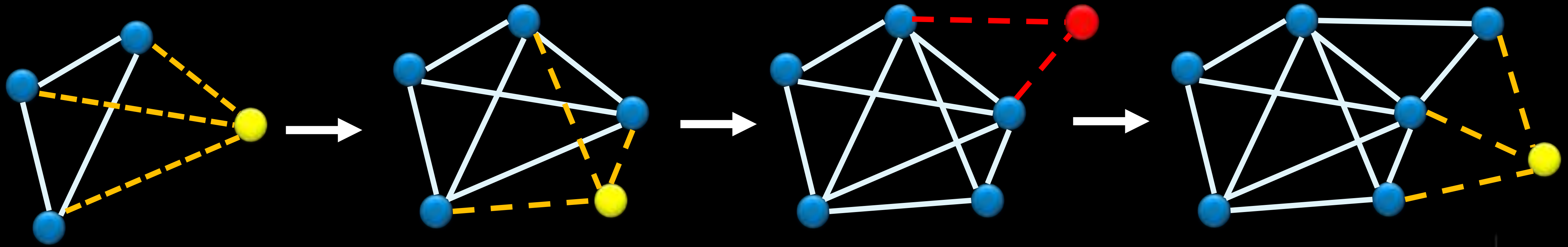
There is a polynomial-time randomized algorithm for checking for generic global rigidity in $\Re^d$.

Algorithms

Representative Algorithms - Trilateration

Incrementally determine all units' location.





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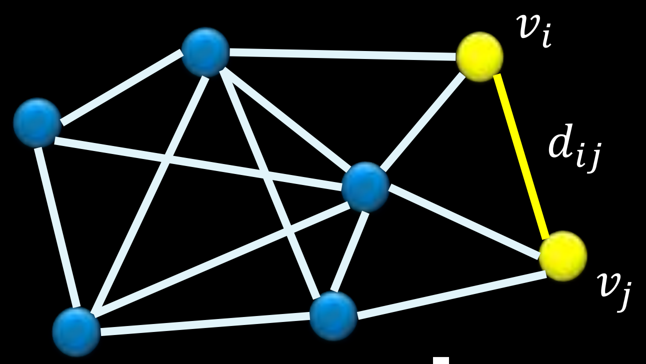
Introduction

Representative Algorithms –

Solving the Distance Equation

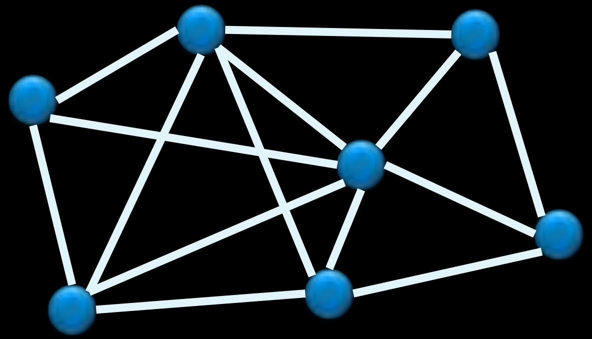
Each edge offers a equation to infer node locations . This is an intuitive way to join together these equations.



$$\left\|P_i - P_j\right\|^2 = d_{ij}^2$$

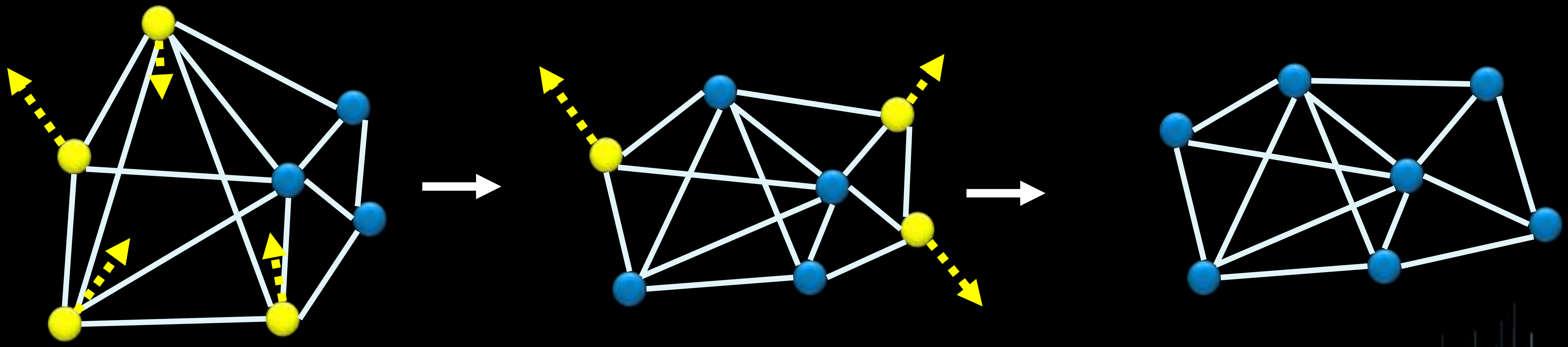
$$\begin{cases} \|P_1 - P_2\|^2 = d_{12}^2 \\ \cdots \\ \left\|P_i - P_j\right\|^2 = d_{ij}^2 \\ \cdots \\ \|P_k - P_n\|^2 = d_{kn}^2 \end{cases}$$

Introduction

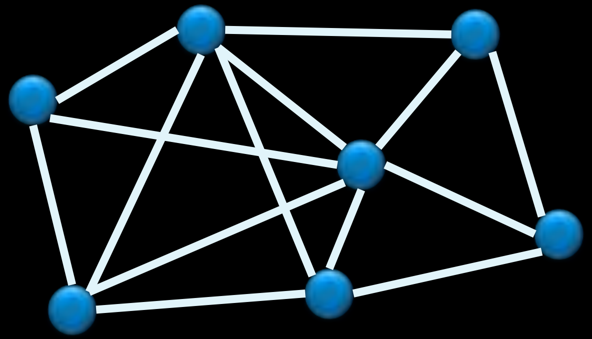


Representative Algorithms - Global Optimization

The gradient of the current result is determined by the objective function. Then improve the accurance of result by iteration.

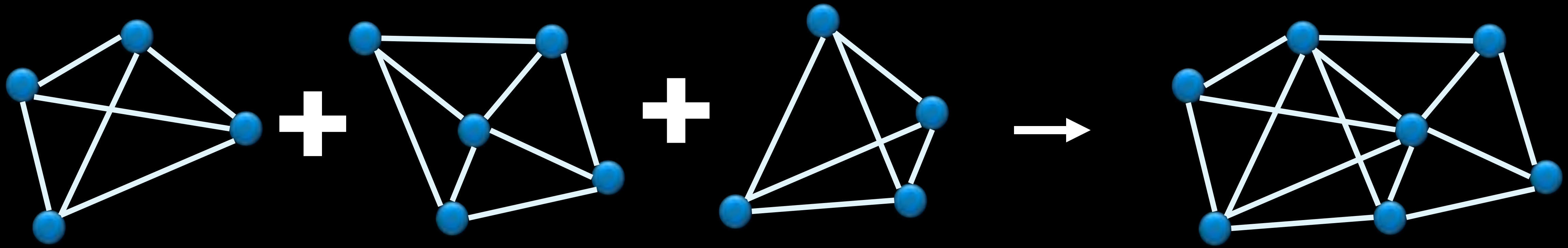


Introduction



Representative Algorithms - Component Stitching

The original graph is divided into some component. The first task is to determine the structurre for each component. Then merge these component toghther.

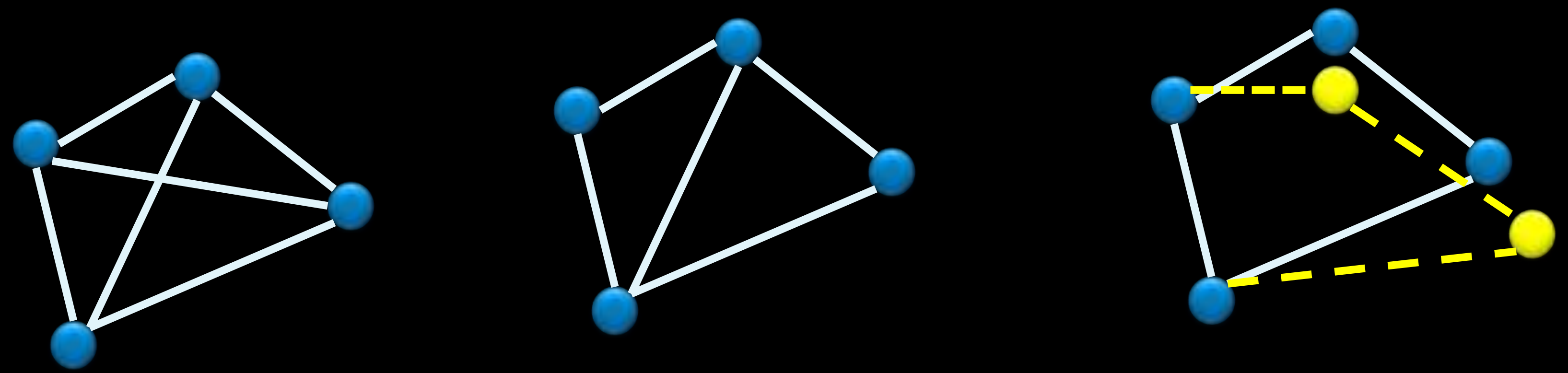


Related Theory

Rigid Theory

1、 A stryucture is rigid if it woundn’t be deformed by the external force;

2、 A graph $G = (V;E)$ with realization in R^2 is rigid if and only if there exists a subset $\varepsilon \subseteq E$ consisting of $|\varepsilon| = 2k - 3$ edges satisfying the property that for any non-empty subset $\varepsilon' \subseteq \varepsilon$, we have $|\varepsilon'| \leq 2k' - 3$, where k’ is the number of nodes in V that are endpoints of $(i, j) \in \varepsilon'$.

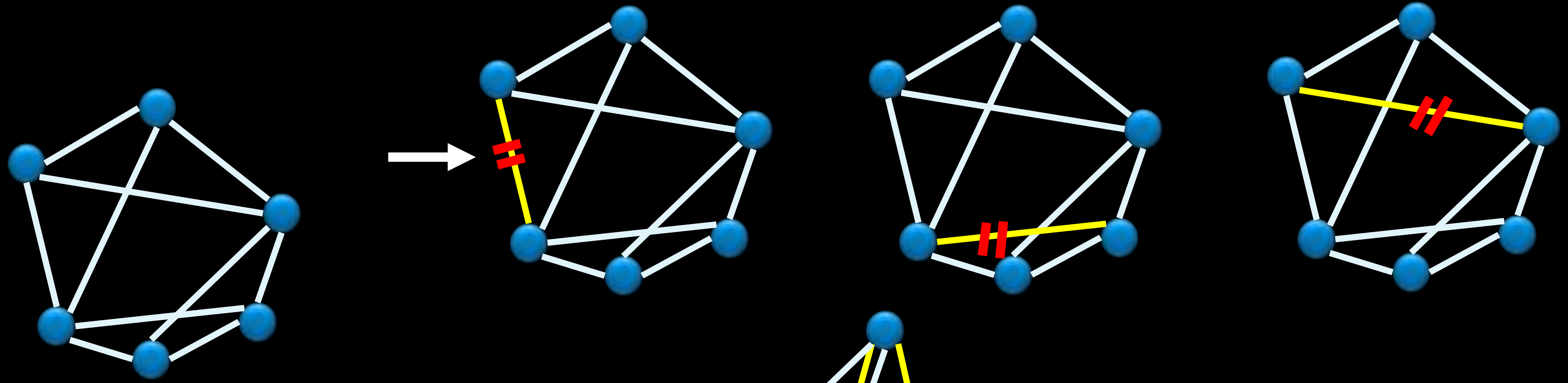


Rigid

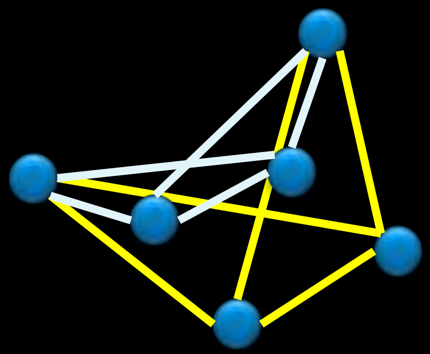
Related Theory

Redundant Rigid

A graph $G = (V;E)$ with realization in R^2 is redundant rigid if and only if it remains rigid after the removal of any edge $(i, j) \in E$.



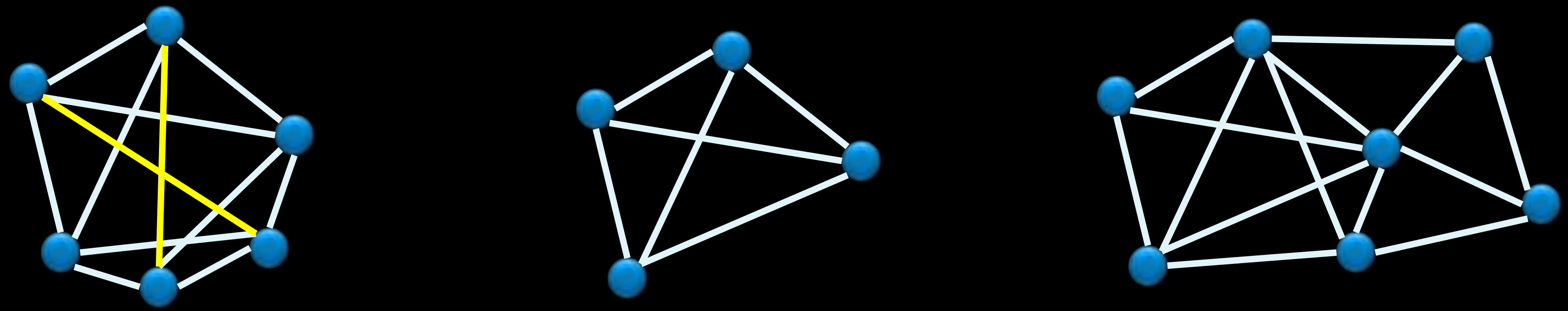
Redundant Rigid



Related Theory

Global Rigid

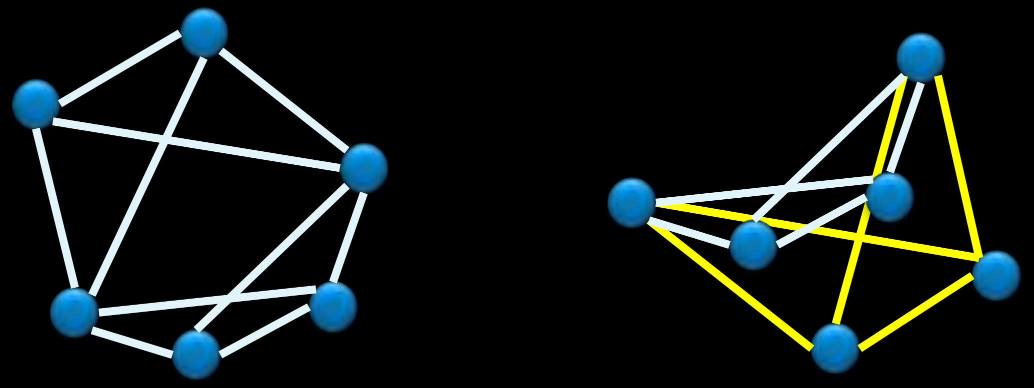
A graph $G = (V;E)$ is global rigid in R^2, then, it's 3-connected and redundant rigid.



Related Theory

Flip and Noise

The realization result may flip due to the strcture of graph. This is realistically disastrous for the results.



The distance obtained in noise results the error in location.

Related Theory

Scalable Graph Realization

Large-Scale Network

This is challenging for computational efficiency and error limited .

Various Network Structure

The units in the network may diferent. And the distribution of nodes may be not uniform, which could result in flipping.

Related Theory

Scalable Graph Realization

Noise

There is noise in most of environment. The distance obtained in noise results the error in location.

Mobile Units

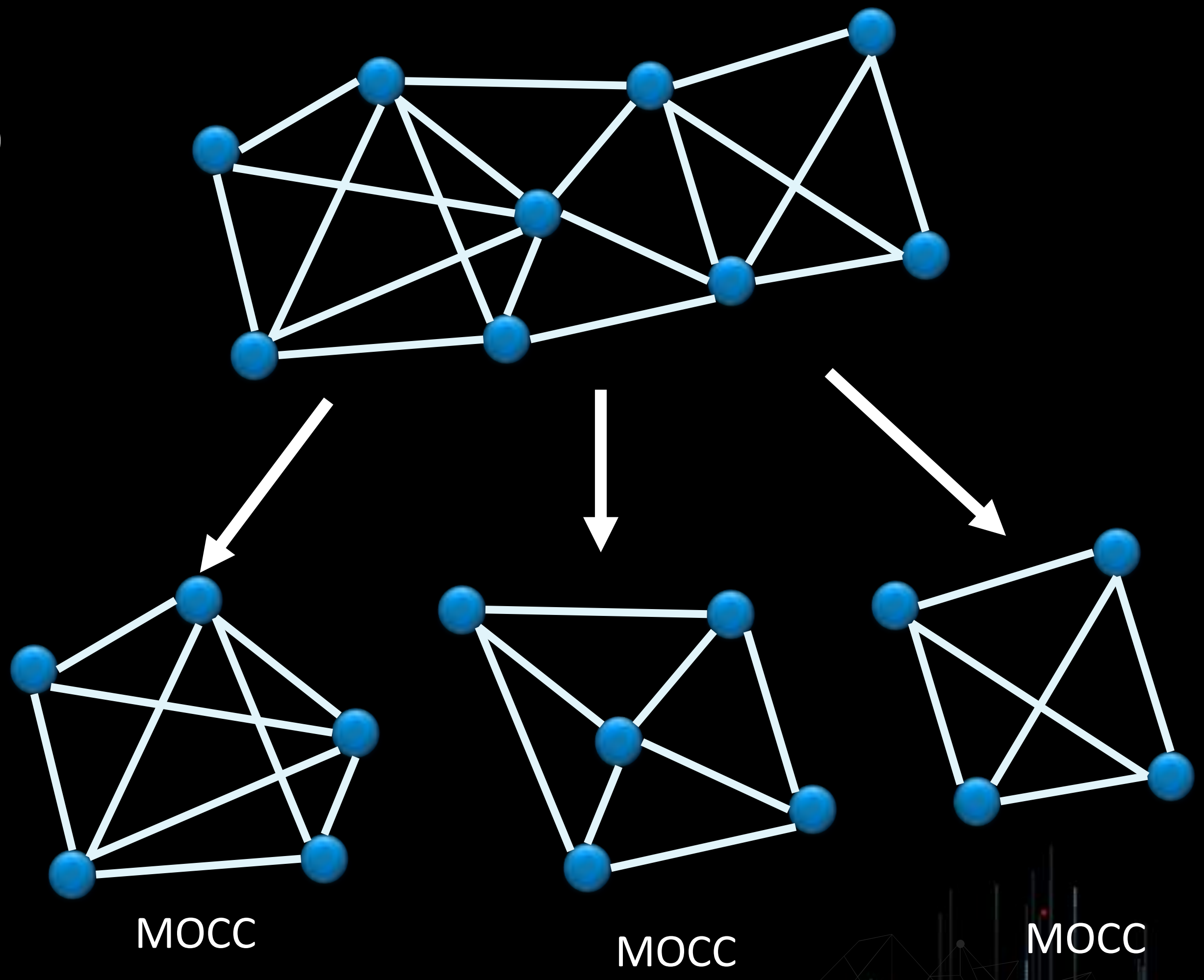
In IoT, some units are mobile. So the structure of network real-time changes.

Practice

1、Most Over Constraint Components(MOCC)

A MOCC is a redundant rigid component; any merged redundant subgraph of it will has smaller redundant ratio than the MOCC.

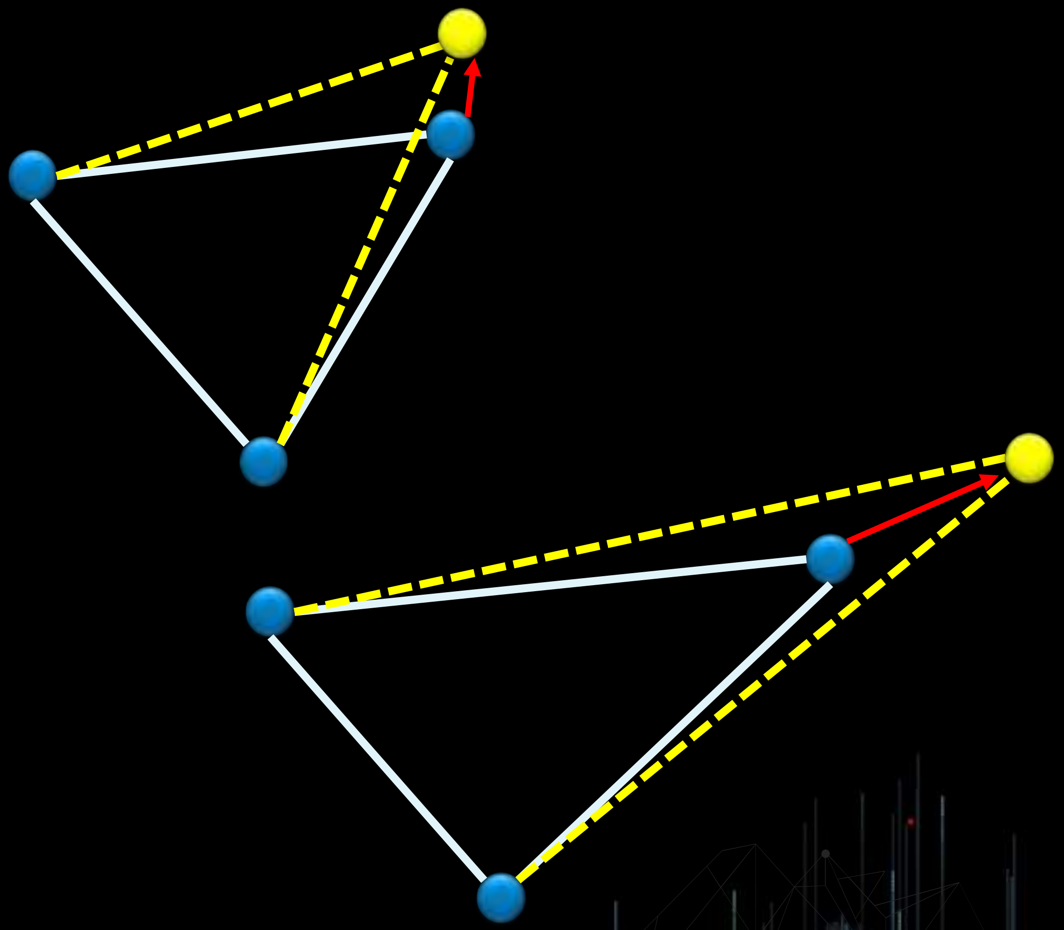
MOCC is a reliable component. Component stitching is one of the most effective method for graph realization problem. It helps to elimate the influence of noise to weight the locally calculated coordinates due to MOCCs, and design an weighted synchronization algorithm for global network coordinate calculation



Practice

2、 Evaluate the Shape of Each Component

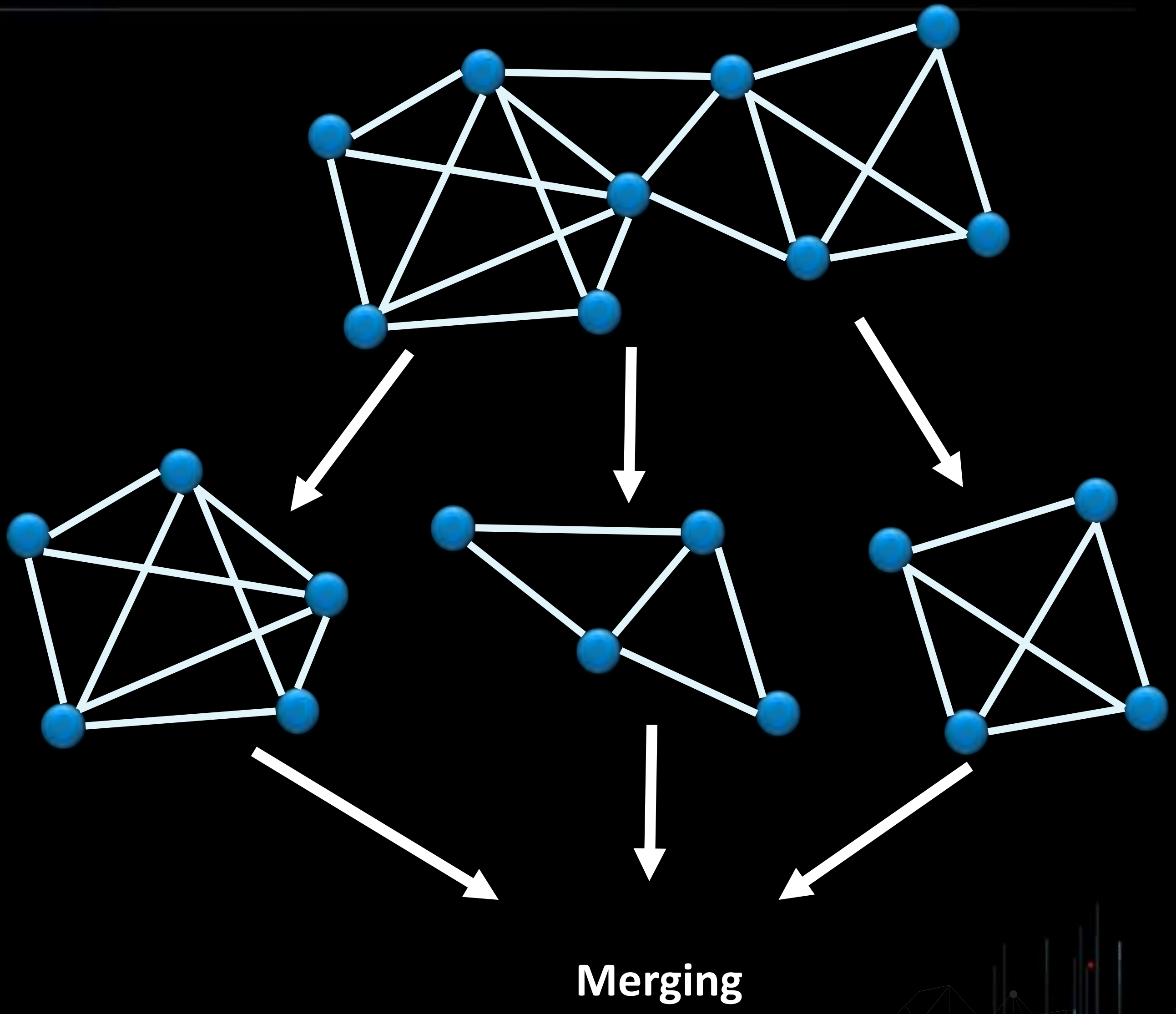
The accuracy of two components' result may be different even if they have the same topology. This is caused by their shape, which influences the ability of anti-noise.



Practice

3、Divide and Rule

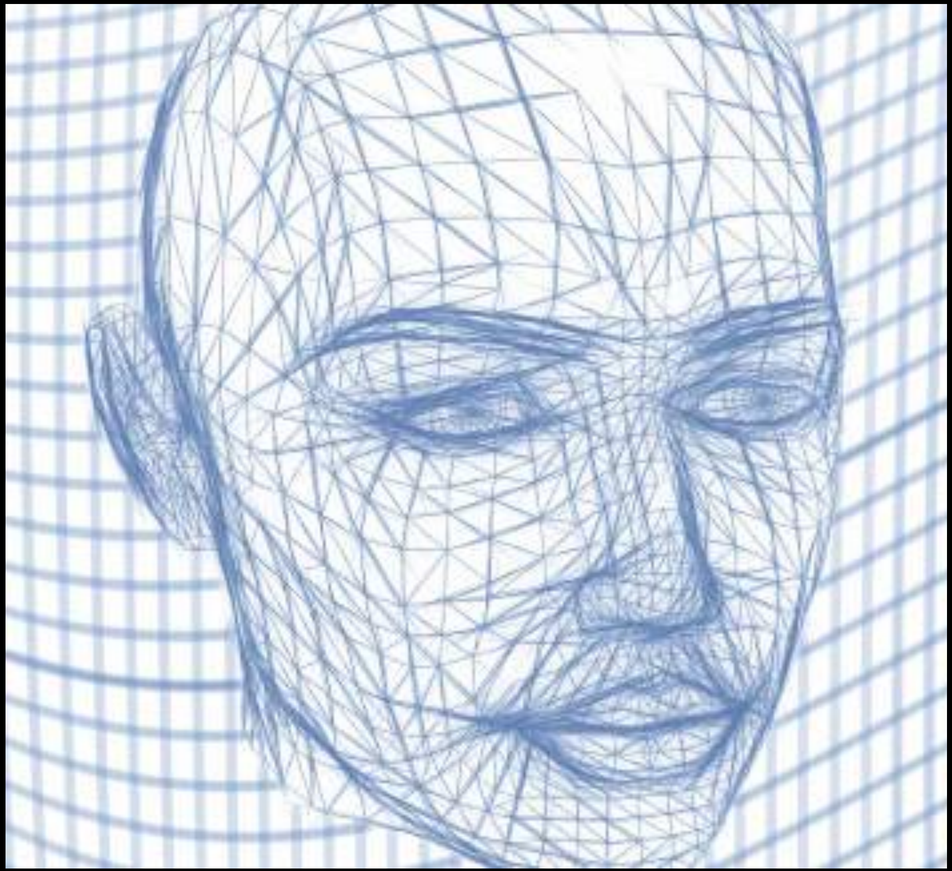
The distribution of nodes is not uniform, and there are some fragile parts in the graph, which may result in flipping. Reconition of these module is important to improve the accuracy of the realization result.



New Trends

Graph Realization in R^3

In SLAM and pattern recognition, it’s a key task of 3D modeling. Graph realization is a basic application to rebuild the surface of objects. The nodes and distances can be obtained by RGB-D camera or other sensors. The we can not only determine each unit’s location in IoT, but also rebuild the surrounding environment with 3D model.



New Trends

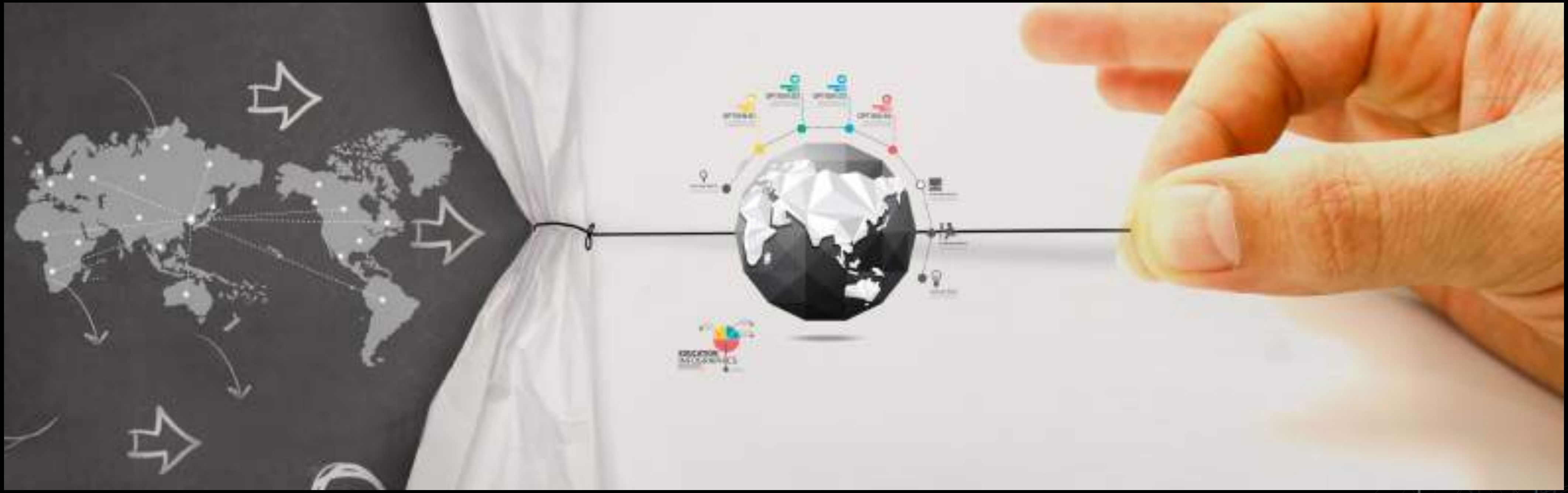
Graph Realization in Large-Scale Network

In smart city, traffic control and environmental monitoring, the scale of network is always so large that it’s challenging for computational efficiency and error limited .

New Trends

Graph Realization in Various Network

IoT is becoming more and more diverse, mobile nodes, various structures and so on. All of these offer platform for graoh realization.



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