Three-Way Decisions and Cognitive Computing

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March 25, 2016

谢谢! Thanks!

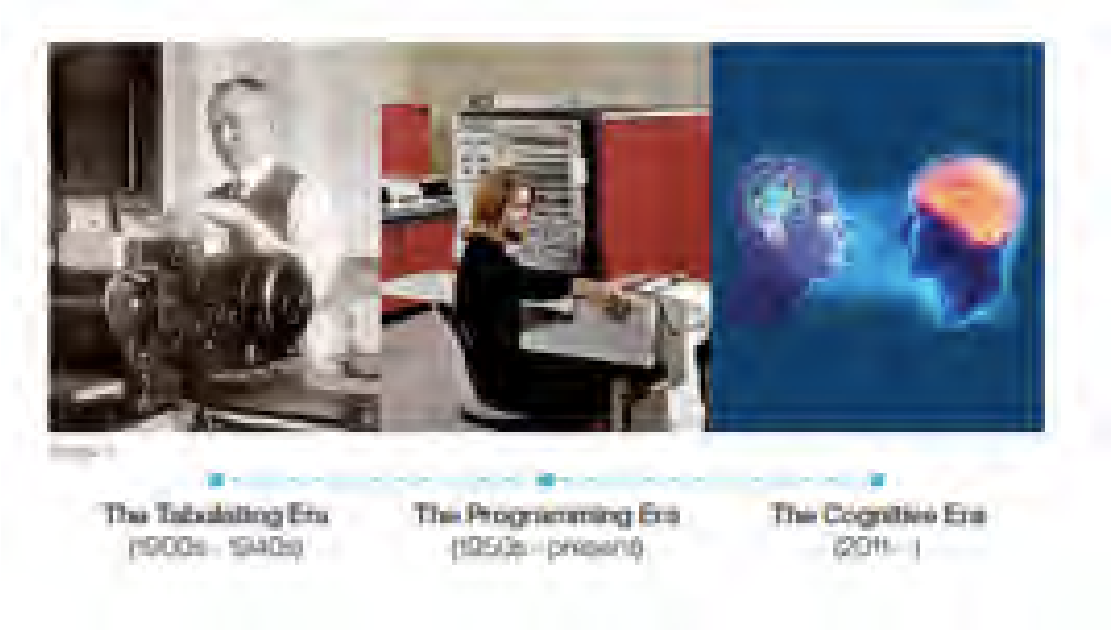
- Professor Guoyin Wang
- Professor Hong Yu
- 2016大数据决策高峰论坛
- Visiting professors and visiting students to the University of Regina
- My wife Guili Liu

Introduction Remarks

- Three-Way Decisions (3WD) are a class of human ways to problem solving and information processing.
- Basic questions:
  - Cognitive basis of 3WD.
  - Cognitive advantages and benefits of 3WD.
  - Human-machine systems based on 3WD.

Introduction Remarks

Three eras of computing:



John E. Kelly III, Computing, cognition and the future of knowing, How humans and machines are forging a new age of understanding, 2015.

Three-Way Decisions

过犹不及。 —《论语·先进》

孔子的学生子贡问孔子他的同学子张和子夏哪个更贤明一些。孔子说子张常常超过周礼的要求，子夏则常常达不到周礼的要求。子贡又问，子张能超过是不是好一些，孔子回答说超过和达不到的效果是一样的。

`http://baike.sogou.com/v112495.htm`

Three-way decisions: 不及, 中, 过

Three-Way Decisions

尽信《书》，则不如无《书》。 — 《孟子·尽心章句下》

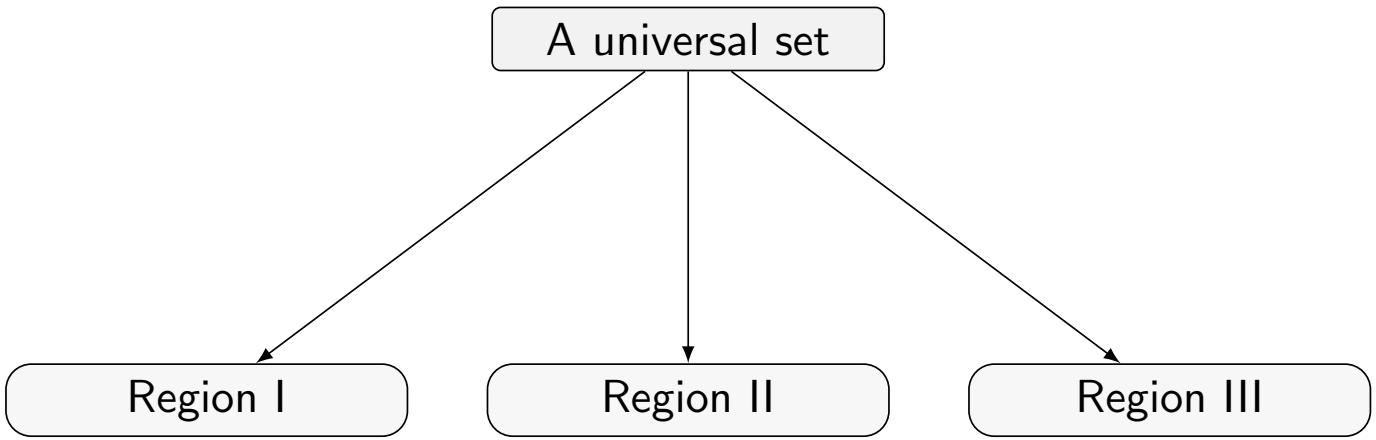
孟子说："完全相信《尚书》，那还不如没有《尚书》。我对于《武成》这一篇书，就只相信其中的二三页罢了。"

http://baike.baidu.com/view/3536804.htm

Three-way decisions: 尽信, 信, 不信 (无书)

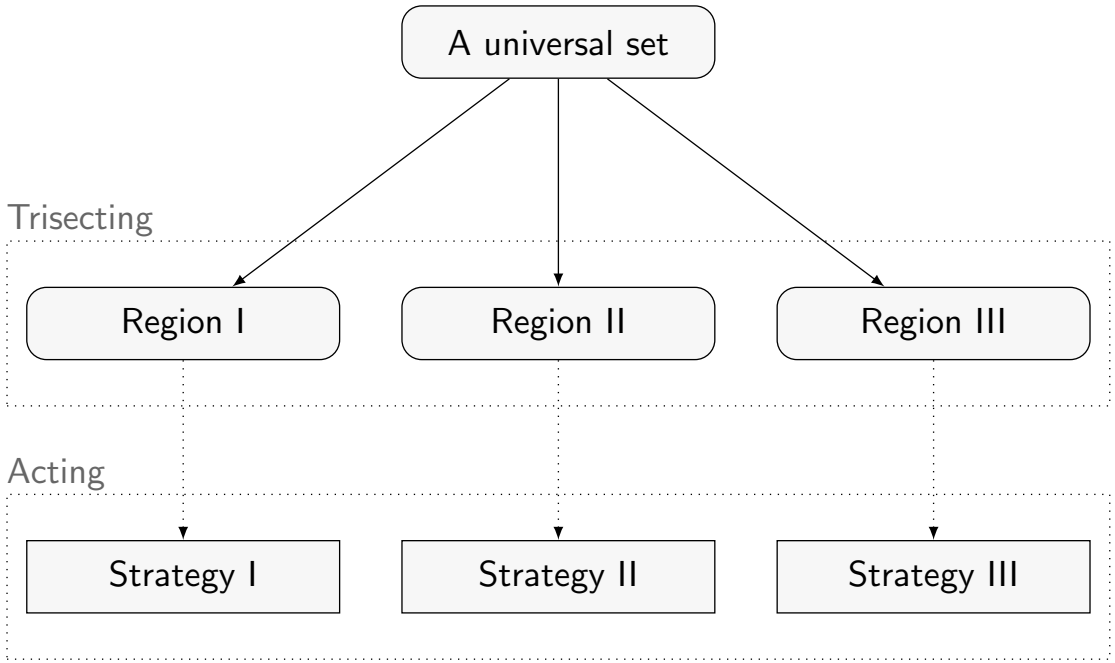
Three-Way Decisions

We divide the whole into three parts. For example, we divide a universal set into three regions:



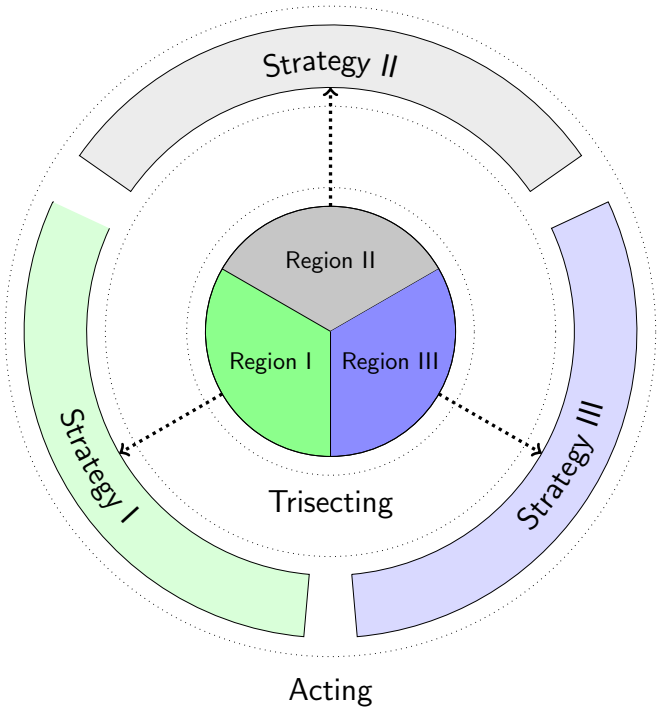
Three-Way Decisions

We use different strategies to process three regions:



姚一豫, 于 洪, 三支决策概述, in: 于洪, 王国胤, 李天瑞, 梁吉业, 苗夺谦, 姚一豫, (编著) 《三支决策：复杂问题求解方法与实践》 科学出版社, 北京, pp. 1-19, 2015.

A Trisecting-and-acting Framework of 3WD



Yao, Y.Y., Rough sets and three-way decisions. RSCTC 2015. LNAI 9436, pp. 62–73, 2015.

Two Basic Components of Three-Way Decisions

- Trisecting: We divide a universal set into three regions.
- Acting: We use different strategies to process three regions

A Broad Meaning of Three-Way Decisions

We may replace "decisions" in "three-way decisions" by other words to have specific interpretations/models:

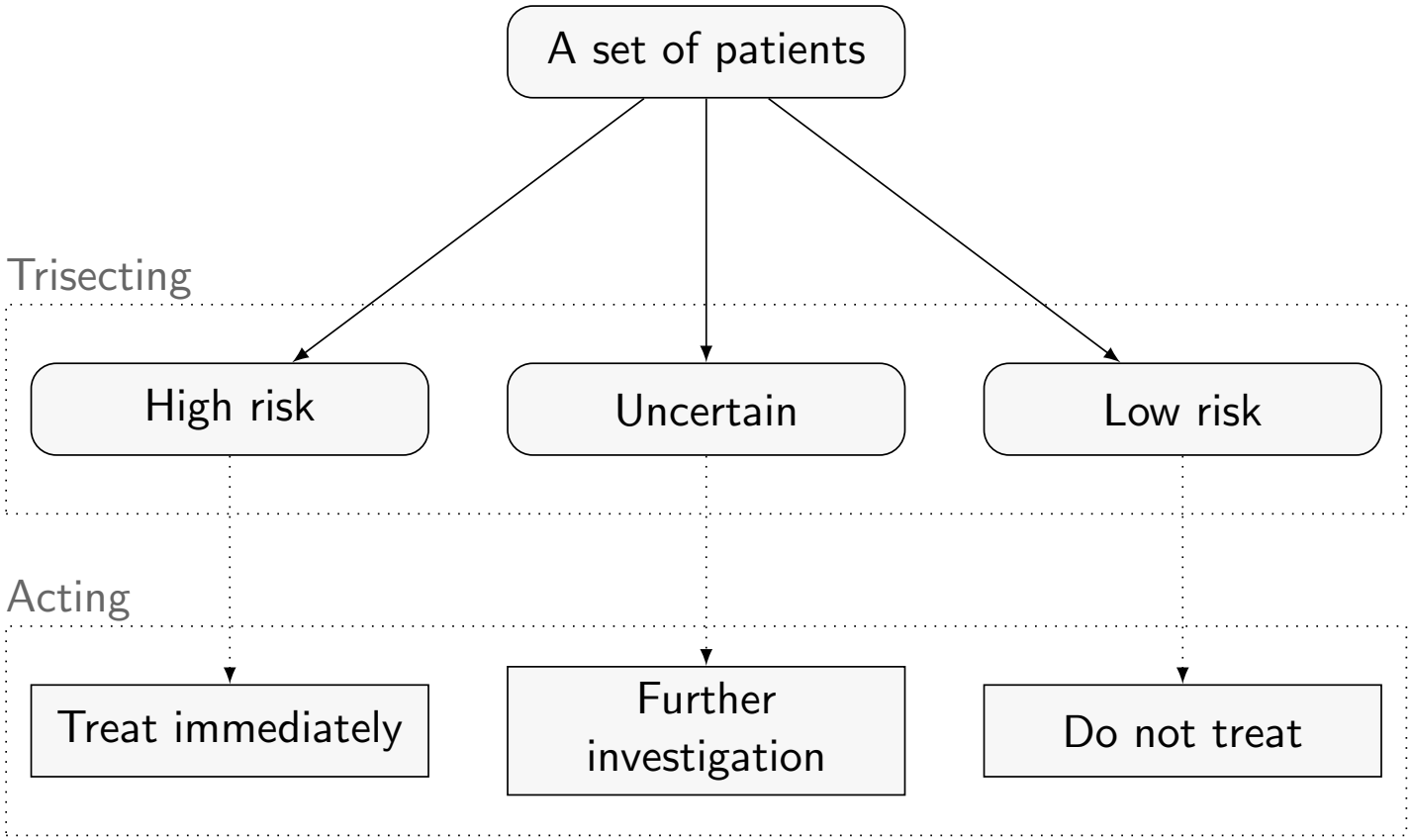
- three-way computing
- three-way processing
- three-way classification
- three-way analysis
- three-way clustering
- three-way recommendation

Y.Y. Yao, Three-way decisions and cognitive computation, 2016

Research Questions

- Is the proposed theory/model reasonable?
- Is the proposed theory/model flexible and general enough?
- Is the proposed theory/model applicable to real-world problems?
- ...
- **Am I on the right track?**

Medical Decisions



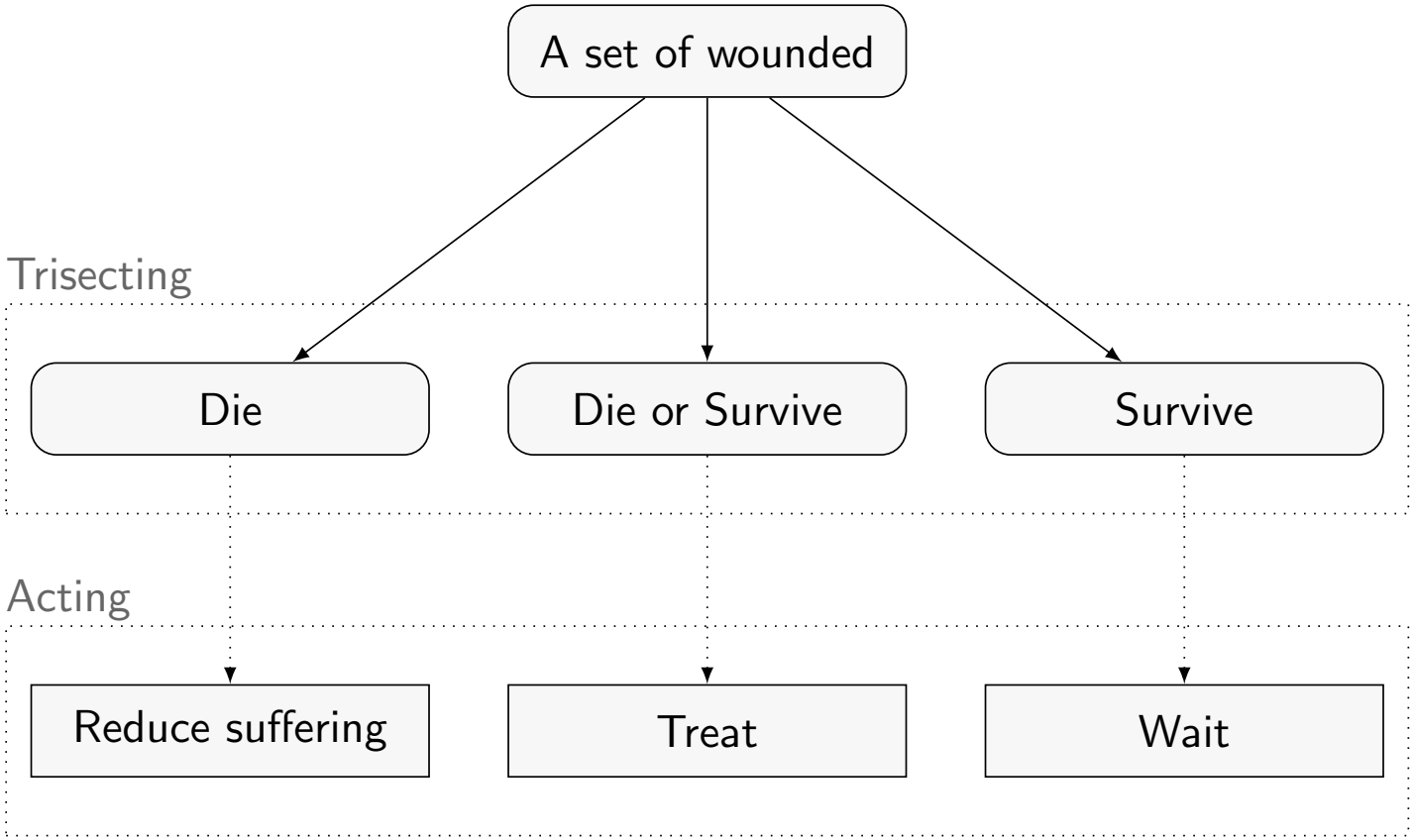
Pauker, S.G., Kassirer, J.P.: The threshold approach to clinical decision making. The New England Journal of Medicine 302, 1109-1117 (1980)

Triage

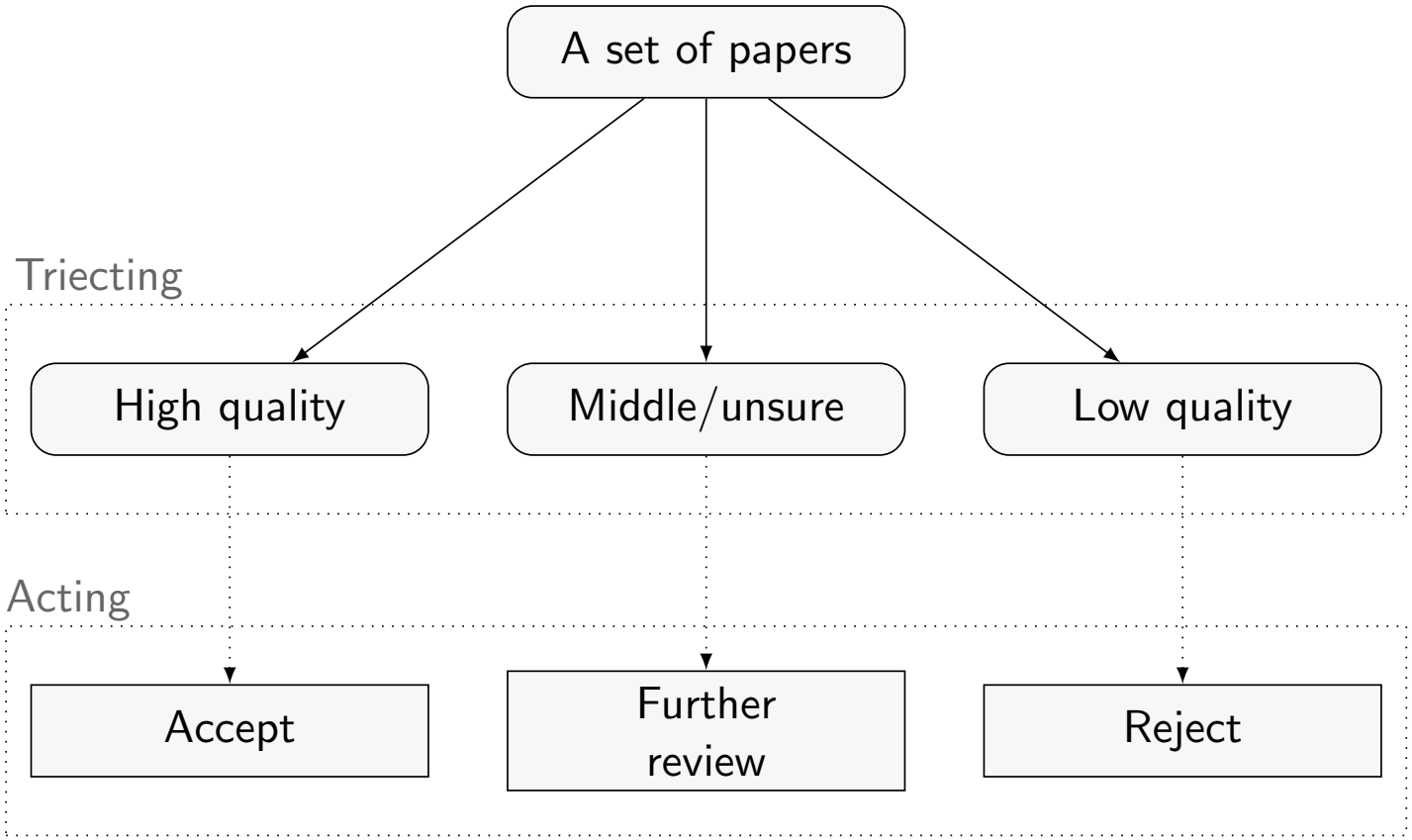
- Triage is the process of determining the priority of patients' treatments based on the severity of their condition.
- Those responsible for the removal of the wounded from a battlefield or their care afterwards would divide the victims into three categories:
  - Those who are likely to live, regardless of what care they receive;
  - Those who are likely to die, regardless of what care they receive;
  - Those for whom immediate care might make a positive difference in outcome.

Wikipedia, Triage, http://en.wikipedia.org/wiki/Triage

Triage

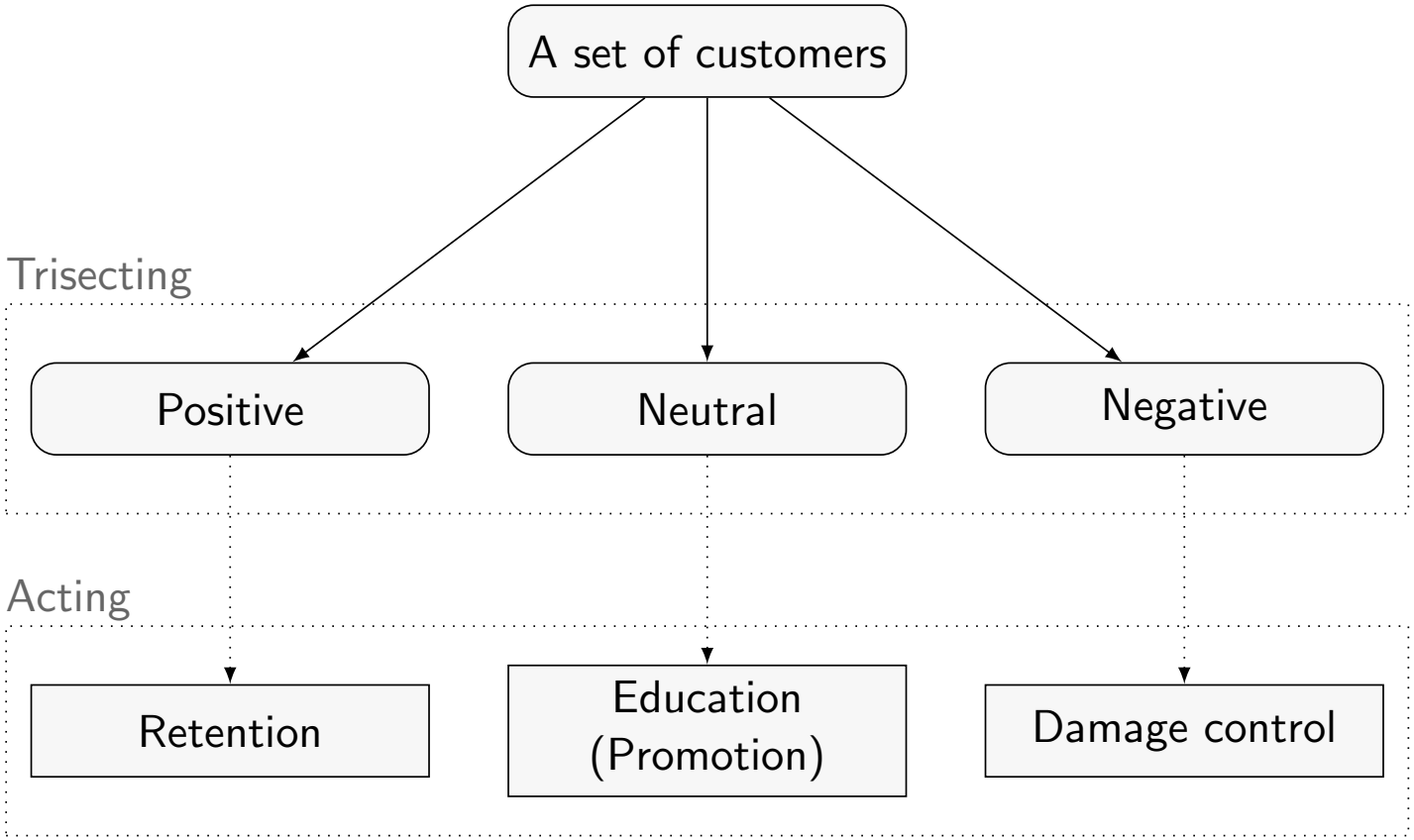

A set of wounded
Trisecting
Die
Die or Survive
Survive
Acting
Reduce suffering
Treat
Wait

Editorial Decision

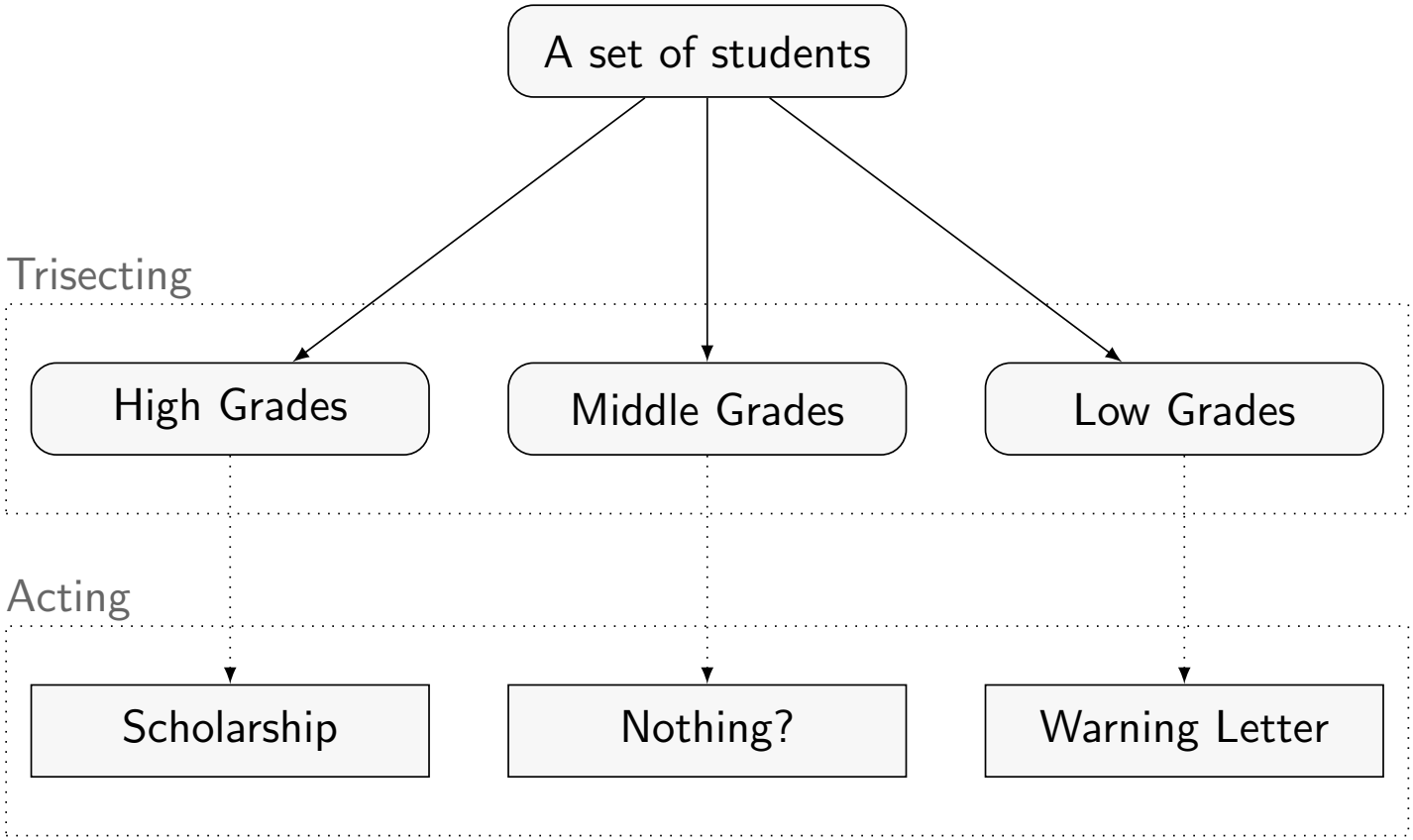


Weller, A.C.: Editorial Peer Review: Its Strengths and Weaknesses. Information Today, Inc., Medford (2001)

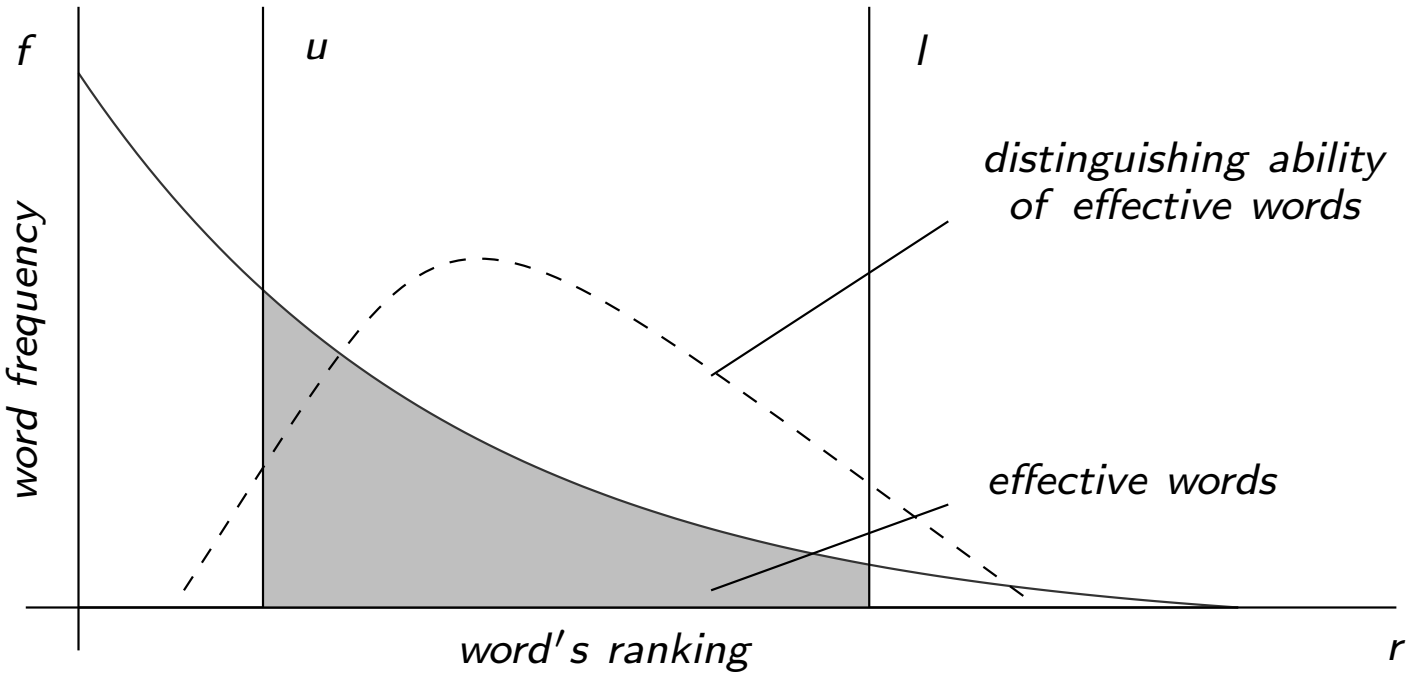
Customer Relation Management



Student Management

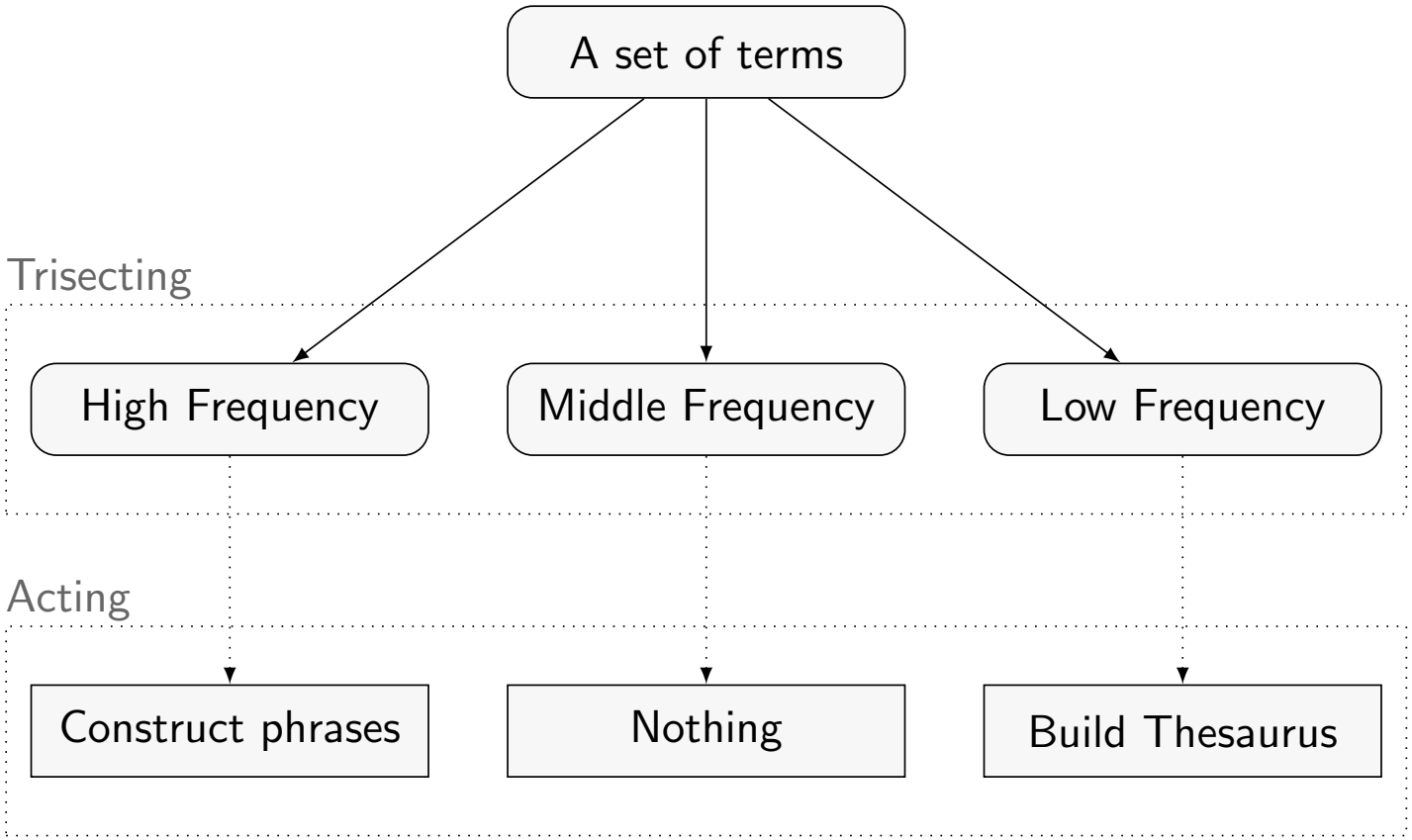


Text Analysis



C.J. van Rijsbergen, Information Retrieval, Butterworth, London, 1979.

Text Analysis



Salton, G,, McGill, M.H. Introduction to modern information retrieval. New York: McGraw-Hill, 1983.

Three-way decisions are everywhere: Temporal (时间)

- yesterday, today, tomorrow
- 昨天, 今天, 明天
- past, present, future
- 过去, 现在, 将来

Three-way decisions are everywhere: Spatial (空间)

- top, middle, bottom,
- 上, 中/不上不下, 下
- front, middle, back
- 前, 中/不前不后, 后
- left, center, right
- 左, 中/不左不右, 右

Three-way decisions are everywhere: Size and volume (尺寸和体积)

- long, medium, short
- 长，不长不短，短
- high, medium, low
- 高, 不高不低, 低
- large, medium, small
- 大, 不大不小, 小

Three-way decisions are everywhere: Attitude (态度)

- positive, neutral, negative
- 正，中性，负
- accept, non-commitment, reject
- 接受，不承诺，拒绝

Three-way decisions are everywhere: Evaluation (评价)

- yes/right, maybe, no/wrong
- 是/对, 可能/不明确, 非/否/错
- upper/top, middle, lower/bottom
- 高, 不高不低, 低
- good, so-so, bad
- 好，不好不坏，坏

Research Questions

- Why the theory is meaningful? (Basis)
- How to build the theory? (Formulation)
- How to apply the theory? (Utility)

Cognitive Basis

- Organization and categorization for simplicity.
- Limited human information processing capacity.
- Evolutionary needs for fast decision-making.

Y.Y. Yao, Three-way decisions and cognitive computing, Cognitive Computation, 2016.

Organization and Categorization

- Humans tend to organize for the sake of simplicity.
- Categorization is essential to mental life.
- Possible results of such organizations are some types of structures.

- **In three-way decisions, we have a tri-partition**.

S. Pinker, How the Mind Works, WW Norton & Company, New York, 1997.

Limited Human Information Processing Capacity

- G.A. Millller (1956), The magical number seven, plus or minus two: Some limits on our capacity for processing information, Psychological Review 101, 343-352.
- N. Cowan (2001), The magical number 4 in short-term memory: A reconsideration of mental storage capacity, Behavioral and Brain Sciences 24, 87-114.

- **The choice of three-way decisions is 3!**

Why 3?

- Three is a magical number.
- Three is a widely used number in human perception, reasoning, and decision-making.

- **The choice of three is appropriate**.

The magical number three in human cognition

- Three is the first number with a "beginning, middle, and end."
- This structure determined by dividing into three is commonly used, for example, in speeches, writings, planning, etc.

Warfield, 1988

Given a tri-partition (A, B, C), we consider

- single regions: A, B, C,
- combinations of two regions: (A, B), (A, C), (B, C)
- combination of three regions: (A, B, C)
- The total number is $3 + 3 + 1 = 7$, which is exactly seven.

If we use four parts, we would have a total of $4 + 6 + 4 + 1 = 15$, which is far more beyond the capacity as suggested by seven.

Warfield JN. The magical number three – plus or minus zero. Cybernetics and Systems 1988; 19: 339-358.

Marketing

Versioning strategies often follow a qualitatively three-part "good-better-best" progression, with increasing number of features and increasing pricing.

Smith T. Pricing strategy: setting price levels, managing price discounts, and establishing price structure. Mason, Ohio: South-Western Cengage Learning; 2012.

Marketing

The optimal number of positive claims is three in order to produce the most positive impression of a product or a service.

Shu SB, Carlson KA. When three charms but four alarms: identifying the optimal number of claims in persuasion settings. Journal of Marketing 2014; 78: 127-139.

Speeches

Lists of three things (e.g., three words, three phrases, or three sentences) are powerful speech patterns commonly used by great speakers.

Clayton M. Brilliant influence: what the most influential people know, do and say. New York: Prentice Hall; 2011.

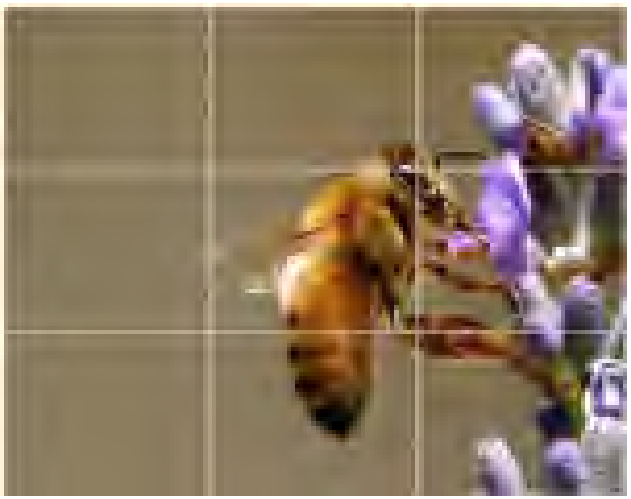
Law

The number three forms psychological bases for standards of decisions.

Clermont KM. Procedure's magical number three: psychological bases for standards of decision. Cornell Law Review 1987; 72: 1115-1156.

Three and other empirical laws

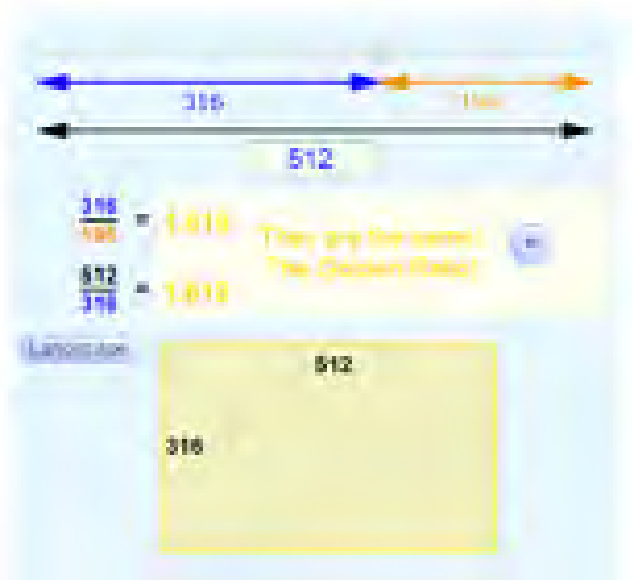
The rule of thirds: to divide a medium into thirds both horizontally and vertically, (0.00-0.33, 0.33-0.67, 0.67-1.00).



http://digital-photography-school.com/rule-of-thirds/

Three and other empirical laws

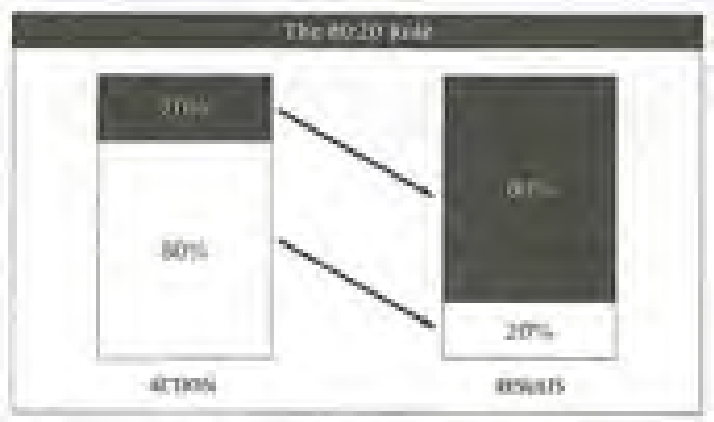
Golden ratio: (0.00-0.382, 0.382-0.618, 0.618-1.00)



http://www.zcool.com.cn/article/ZMTcyNTI0.html

Three and other empirical laws

Pareto Principle or 20/80 Laws: (20%, 60%, 20%)



http://musicianguide.cn/8020-rule-in-music-social-media-marketing?utm_source=weixin&utm_medium=mgarticle&utm_campaign=socialmedia

Evolutionary Needs for Fast Decision-Making

Two modes of thinking, two systems in the mind:

- System 1 operates automatically and quickly, with little or no effort and no sense of voluntary control.
- System 2 allocates attention to the effortful mental activities that demand it, including complex computations.

- **Some regions are suitable for system 1 and some for system 2**.

D. Kahneman, Thinking, Fast and Slow, Anchor Canada, 2013.

Three-Way Decisions: Cognitive Advantages

- Three regions correspond to two extreme points (i.e., the two polarities) and a middle point.
- Humans can easily determine and process objects that are instances of the two extreme cases.
- Three presents a small number of regions that are easy to manage.

Three-Way Decisions: Benefits

- Trade-off between accuracy and cost.
- Trade-off between effectiveness and efficiency.
- Trade-off between benefits of the majority and the benefits of a few.
- Trade-off between risk of immediately decisions and deferred decisions.

Research Questions

- How to determine the three regions?
- How to effectively process each region?

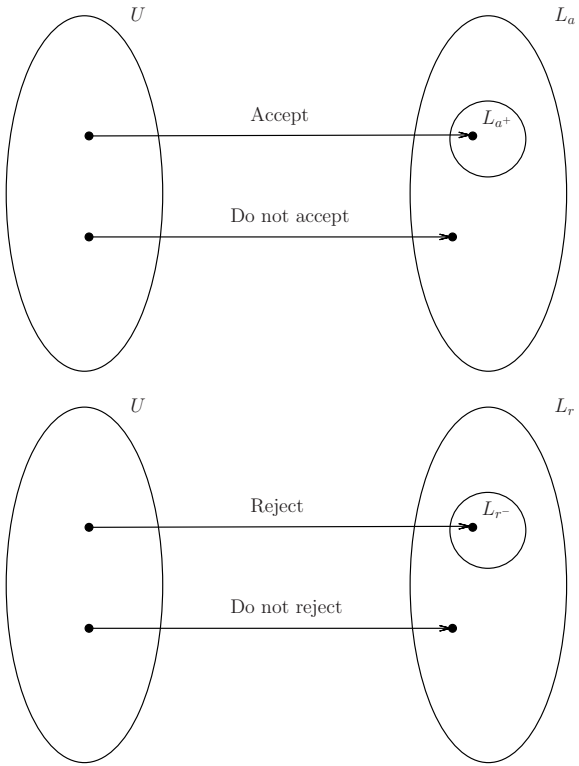
- **A theory of three-way decisions can be formally developed**.

Y.Y. Yao, An Outline of a Theory of Three-way Decisions, RSCTC 2012, LNCS (LNAI) 7413, pp. 1-17, 2012.

Basic Ideas

- Build evaluation functions for dividing objects into three regions:
  - Determine and interpret the values of evaluation functions for three-way decisions.
  - Determine three regions based on evaluation status values.
- Utilize the three regions, that is, process the three regions by designing best strategies.

Model 1: A Pair of Poset-based Evaluations

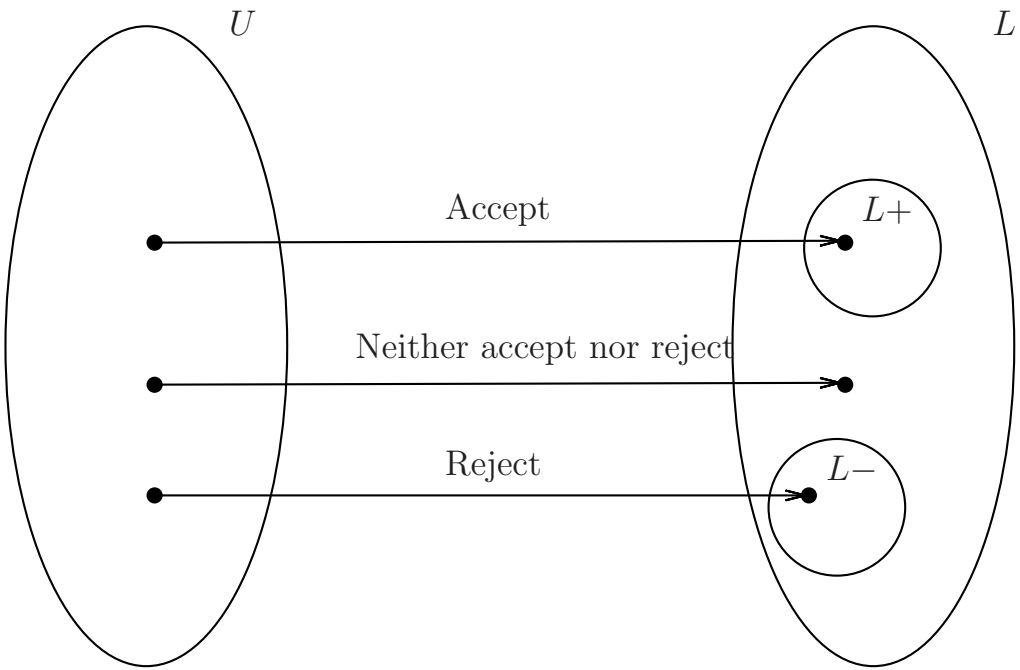


Model 1: A Pair of Poset-based Evaluations

The three regions are defined by:

$$
\begin{aligned}
\mathrm{L}_{(L_a^+, L_r^-)}(v_a, v_r) &= \{x \in U \mid v_a(x) \in L_a^+ \wedge v_r(x) \notin L_r^-\}, \\
\mathrm{R}_{(L_a^+, L_r^-)}(v_a, v_r) &= \{x \in U \mid v_a(x) \notin L_a^+ \wedge v_r(x) \in L_r^-\}, \\
\mathrm{M}_{(L_a^+, L_r^-)}(v_a, v_r) &= (\mathrm{POS}_{(L_a^+, L_r^-)}(v_a, v_r) \cup \mathrm{NEG}_{(L_a^+, L_r^-)}(v_a, v_r))^c \\
&= \{x \in U \mid (v_a(x) \notin L_a^+ \wedge v_r(x) \notin L_r^-) \vee \\
&\qquad (v_a(x) \in L_a^+ \wedge v_r(x) \in L_r^-)\}.
\end{aligned}
$$

Model 2: One Poset-based Evaluation

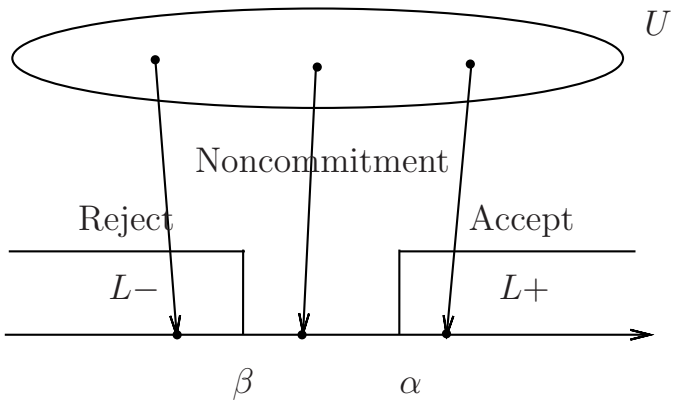


Model 2: One Poset-based Evaluation

The three regions are defined by:

$$\begin{aligned}
\mathrm{L}_{(L^+,L^-)}(v) &= \{x \in U \mid v(x) \in L^+\},\\
\mathrm{R}_{(L^+,L^-)}(v) &= \{x \in U \mid v(x) \in L^-\},\\
\mathrm{M}_{(L^+,L^-)}(v) &= \{x \in U \mid v(x) \notin L^+ \wedge v(x) \notin L^-\}.
\end{aligned}$$

Model 3: A Totally Ordered Set based Evaluation



Model 3: A Totally Ordered Set based Evaluation

The three regions are defined by:

$$\begin{aligned}
\mathrm{L}_{(\alpha,\beta)}(v) &= \{x \in U \mid v(x) \succeq \alpha\},\\
\mathrm{R}_{(\alpha,\beta)}(v) &= \{x \in U \mid v(x) \preceq \beta\},\\
\mathrm{M}_{(\alpha,\beta)}(v) &= \{x \in U \mid \beta \prec v(x) \prec \alpha\}.
\end{aligned}$$

Construction of Evaluations (An Example)

Suppose $C = \{c_1, c_2, \ldots, c_m\}$ are a set of m criteria. Suppose $v_{c_i} : U \longrightarrow \Re$ denotes an evaluation based on criterion v_i, $1 \leq i \leq m$. An overall evaluation function $v : U \longrightarrow \Re$ may be simply defined by a linear combination of individual evaluations:

$$v(x) = w_1 v_{c_1}(x) + w_2 v_{c_2}(x) + \ldots + w_m v_{c_m}(x).$$

Connection to multi-criteria decision making (MCDM)

Determination of Designated Sets of Values (An Example)

Let $R_L(\alpha, \beta)$, $R_R(\alpha, \beta)$ and $R_M(\alpha, \beta)$ denote the risks of the three regions, respectively. It is reasonable to require that the sets of designated values are chosen to minimize the following overall risks:

$$R(\alpha, \beta) = aR_L(\alpha, \beta) + bR_M(\alpha, \beta) + cR_R(\alpha, \beta).$$

$$\arg \min_{(\alpha,\beta)} R(\alpha, \beta).$$

Connection to multi-objective decision making (MODM)

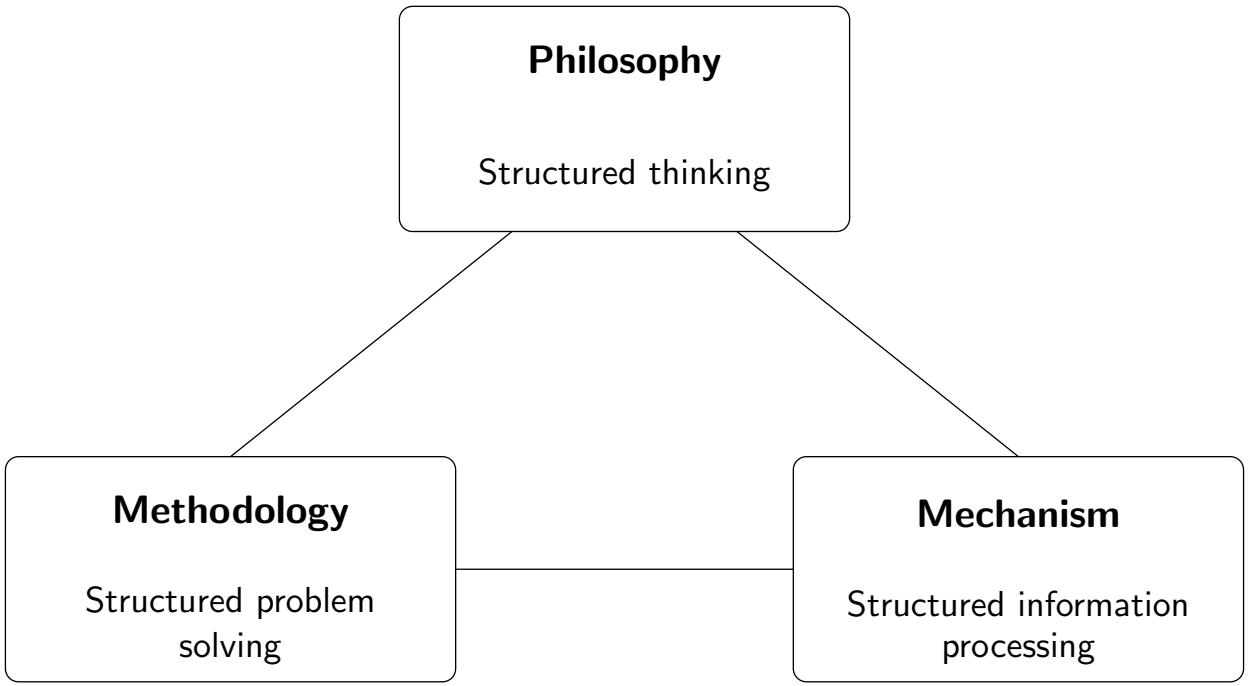
A Triarchic Theory of Granular Computing

Granular computing is an emerging field of study that explores different levels of granularity in thinking, problem solving, and information processing.

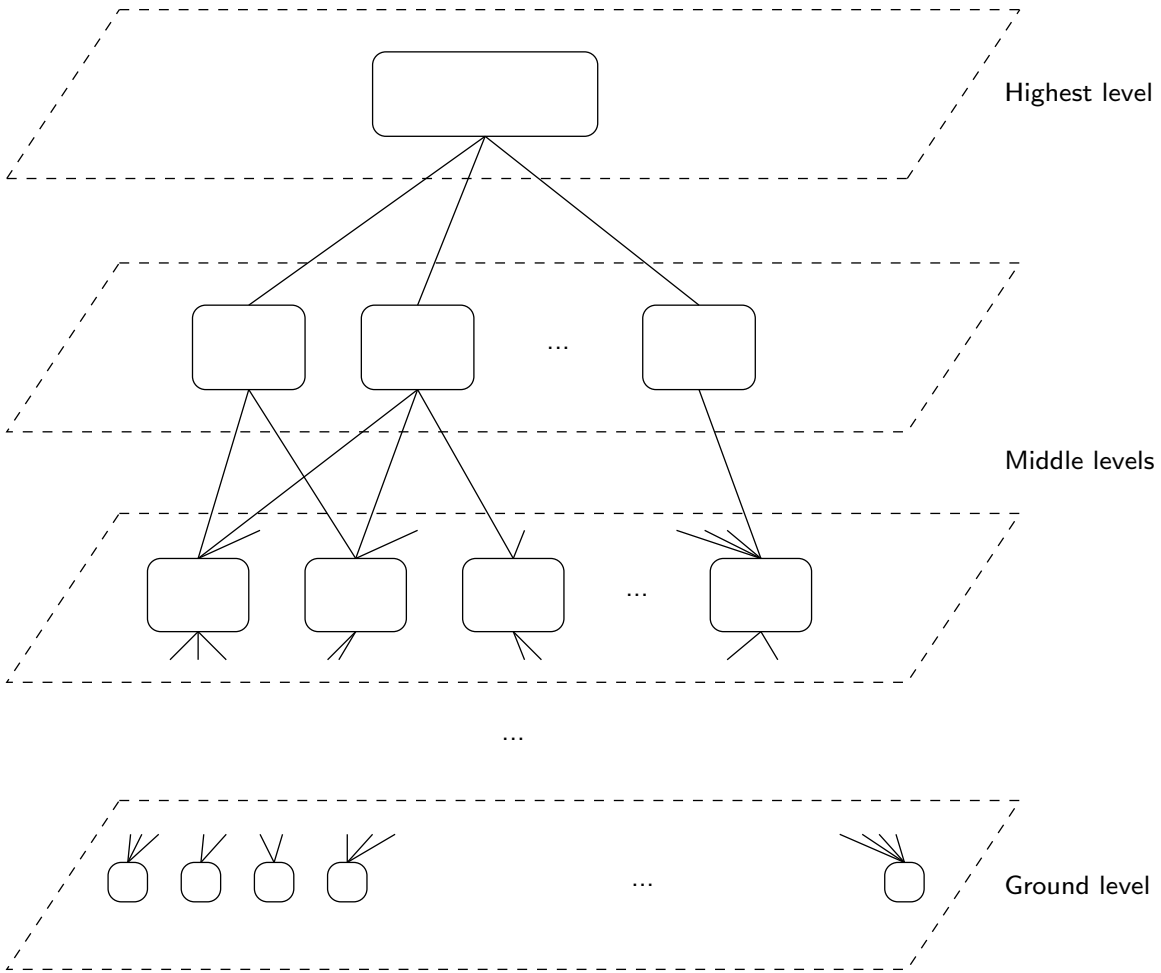
Y.Y. Yao, A triarchic theory of granular computing, Granular Computing, 2016.

http://link.springer.com/article/10.1007/s41066-015-0011-0

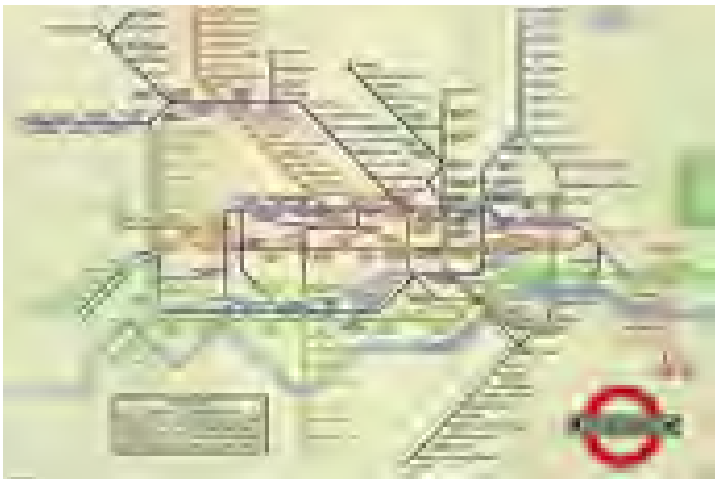
The granular computing triangle



Granular structures



An example



Main Ideas

- Focus on the essential of the problem.
- Not distracted by irrelevant details.

Basic principles of sequential three-way decisions

"··· examine the problem at a finer granulation level with more detailed information when there is a need or benefit for doing so."

Yao, Y.Y.: Granular computing: Basic issues and possible solutions. In: Proceedings of the 5th Joint Conference on Information Sciences, vol. 1, pp. 186–89 (2000)

Basic principles of sequential three-way decisions

make a decision "at a finer granulation level with more detailed information when there is a need or benefit for doing so."

Yao, Y.Y.: Granular Computing and Sequential Three-Way Decisions. RSKT 2013, LNCS (LNAI), vol. 8171, pp. 16-27

Basic ingredients of sequential TWD

- Multiple levels of granularity.
- Multiple descriptions of objects.
- Multiple evaluations of objects.
- Three-way decisions at a particular level.
- Sequential three-way decisions at multiple levels.

Multiple levels of granularity.

- There are $n+1$, $n \geq 1$, levels of granularity. For simplicity, we use the index set $\{0, 1, 2, \ldots, n\}$ to denote the $n+1$ levels, with 0 representing the finest granularity (i.e., the ground level) and n the coarsest granularity.

Multiple levels of granularity

- There are $n+1$, $n \geq 1$, levels of granularity. For simplicity, we use the index set $\{0, 1, 2, \ldots, n\}$ to denote the $n+1$ levels, with 0 representing the finest granularity (i.e., the ground level) and n the coarsest granularity.

Multiple descriptions of objects

- We have $n+1$ distinct representations and descriptions of the same object at different levels. Suppose

$$\mathrm{Des}_0(x) \preceq \mathrm{Des}_1(x) \preceq \ldots \preceq \mathrm{Des}_n(x), \tag{1}$$

  is a sequence of descriptions of object $x \in U$ with respect to $n+1$ levels of granularity.
- The relation $\preceq$ denotes a "finer than" relationship between different descriptions. There are $n+1$, $n \geq 1$, levels of granularity.

Multiple evaluations of objects

- Let v_i, $0 \leq i \leq n$, denote an evaluation at level i whose values are from a totally ordered sets $(L_i, \preceq_i)$.

Three-way decisions at a particular level

- Suppose U_{i+1} is the set of objects with a non-commitment decision from level $i+1$.
- For level n, we use the entire set U as the set of objects with a non-commitment decision, i.e., $U_{n+1} = U$.
- For level i, $1 \leq i \leq n$, we can choose a pair of thresholds $\alpha_i, \beta_i \in L_i$ with $\beta_i \prec_i \alpha_i$. Three-way decision making can be expressed as:

$$\begin{aligned}
\mathrm{POS}_{(\alpha_i,\beta_i)}(v_i) &= \{x \in U_{i+1} \mid v_i(\mathrm{Des}_i(x)) \succeq_i \alpha_i\}, \\
\mathrm{NEG}_{(\alpha_i,\beta_i)}(v_i) &= \{x \in U_{i+1} \mid v_i(\mathrm{Des}_i(x)) \preceq_i \beta_i\}, \\
\mathrm{BND}_{(\alpha_i,\beta_i)}(v_i) &= \{x \in U_{i+1} \mid \beta_i \prec_i v_i(\mathrm{Des}_i(x)) \prec_i \alpha_i\}. \quad (2)
\end{aligned}$$

- The boundary region gives set of objects with a non-commitment decision, namely, $U_i = \mathrm{BND}_{(\alpha_i,\beta_i)}(v_i)$.

Sequential TWD

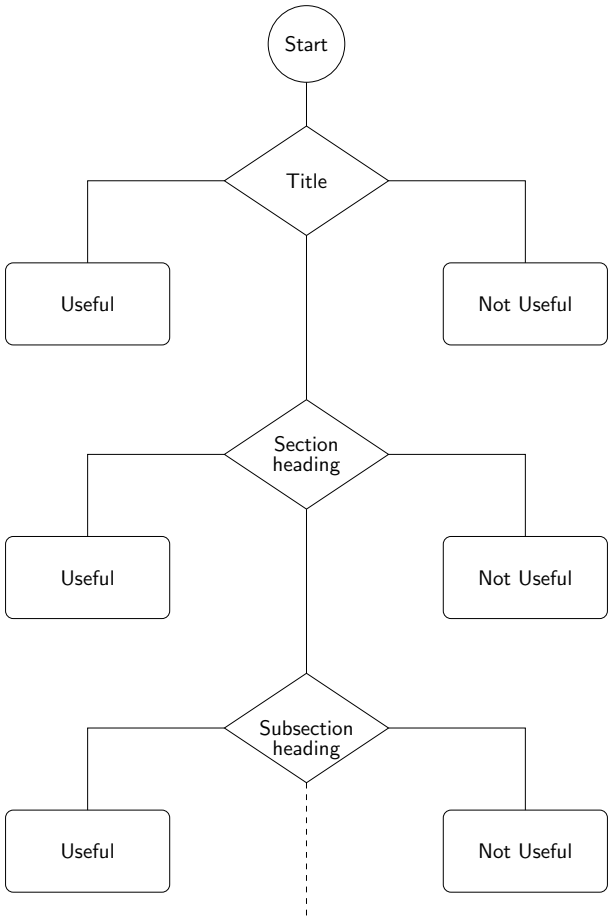
Input: A set of objects U, a family of descriptions for each object $\{\mathrm{Des}_i(x)\}$, a set of evaluations $\{v_i\}$, and a set of pairs of thresholds $\{(\alpha_i, \beta_i)\}$;
Output: Two regions POS and NEG;

begin
  $\mathrm{POS} = \emptyset$;
  $\mathrm{NEG} = \emptyset$;
  $i = n$;
  $U_{n+1} = U$;
  $U_1 = \emptyset$;
  while $U_{i+1} \neq \emptyset$ and $i > 0$ **do**
    $\mathrm{POS}_{(\alpha_i,\beta_i)}(v_i) = \{x \in U_{i+1} \mid v_i(\mathrm{Des}_i(x)) \succeq_i \alpha_i\}$;
    $\mathrm{NEG}_{(\alpha_i,\beta_i)}(v_i) = \{x \in U_{i+1} \mid v_i(\mathrm{Des}_i(x)) \preceq_i \beta_i\}$;
    $\mathrm{BND}_{(\alpha_i,\beta_i)}(v_i) = \{x \in U_{i+1} \mid \beta_i \prec_i v_i(\mathrm{Des}_i(x)) \prec_i \alpha_i\}$;
    $\mathrm{POS} = \mathrm{POS} \cup \mathrm{POS}_{(\alpha_i,\beta_i)}(v_i)$;
    $\mathrm{NEG} = \mathrm{NEG} \cup \mathrm{NEG}_{(\alpha_i,\beta_i)}(v_i)$;
    $U_i = \mathrm{BND}_{(\alpha_i,\beta_i)}(v_i)$;
    $i = i - 1$;
  if $U_1 \neq \emptyset$ **then**
    $\mathrm{POS}_{\gamma_0}(v_0) = \{x \in U \mid v_0(\mathrm{Des}_0(x)) \succeq \gamma_0\}$;
    $\mathrm{NEG}_{\gamma_0}(v_0) = \{x \in U \mid v_0(\mathrm{Des}_0(x)) \prec \gamma_0\}$;
    $\mathrm{POS} = \mathrm{POS} \cup \mathrm{POS}_{\gamma_0}(v_0)$;
    $\mathrm{NEG} = \mathrm{NEG} \cup \mathrm{NEG}_{\gamma_0}(v_0)$;
  return POS, NEG;

An Example of Multilevel Granular Structure

- Scientific paper:
  - Title
  - Section headings
  - Subsection headings
  - Abstract
  - Introduction + conclusion
  - Full paper

Reading as S3WD

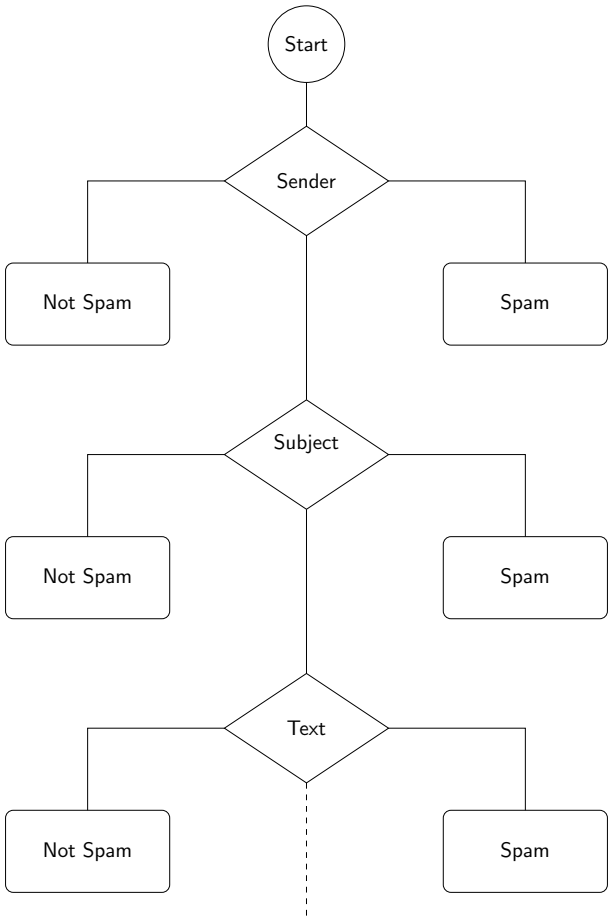


Write your paper carefully so other researchers will read and find it useful.

Another Example of Multilevel Granular Structure

- Email:
  - Sender
  - Subject
  - Text
  - Attachments

Spam Filtering as S3WD



J.L. Li, X.F. Deng, Y.Y. Yao, Multistage email spam filtering based on three-way decisions, RSKT 2013, 313-324, 2013.

Three-Way Decisions and Granular Big Data

- Application of multiple levels of description of data and multiple data-processing and data-analyzing models.
- Three-way selection of useful data:
  - Useful
  - Possibly useful
  - Non-useful
- Sequential three-way granular data analytics:
  - Multilevel analysis
  - Data selection
  - Model selection

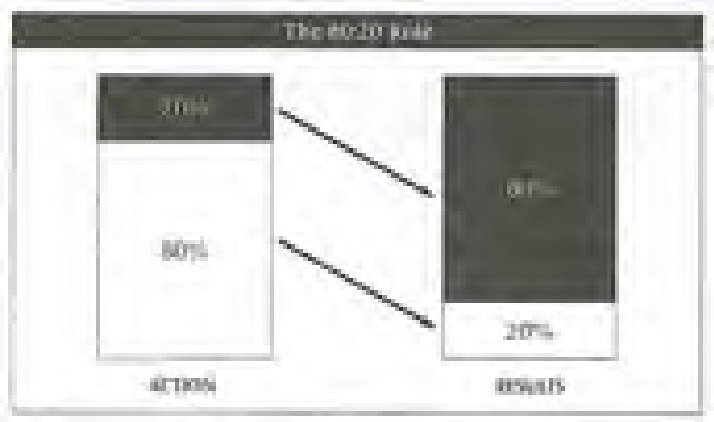
Three-way Data Selection

Pareto principle (also known as the 80–20 rule, the law of the vital few, and the principle of factor sparsity):

- for many events, roughly 80% of the effects come from 20% of the causes:
  - 80% of a company's profits come from 20% of its customers.
  - 80% of a company's sales come from 20% of its products.
  - 20% the code has 80% of the errors.
  - 20% of the books are borrowed 80% of the time.
- 20% of the data are sufficient 80% of the time.
- How to select useful data?

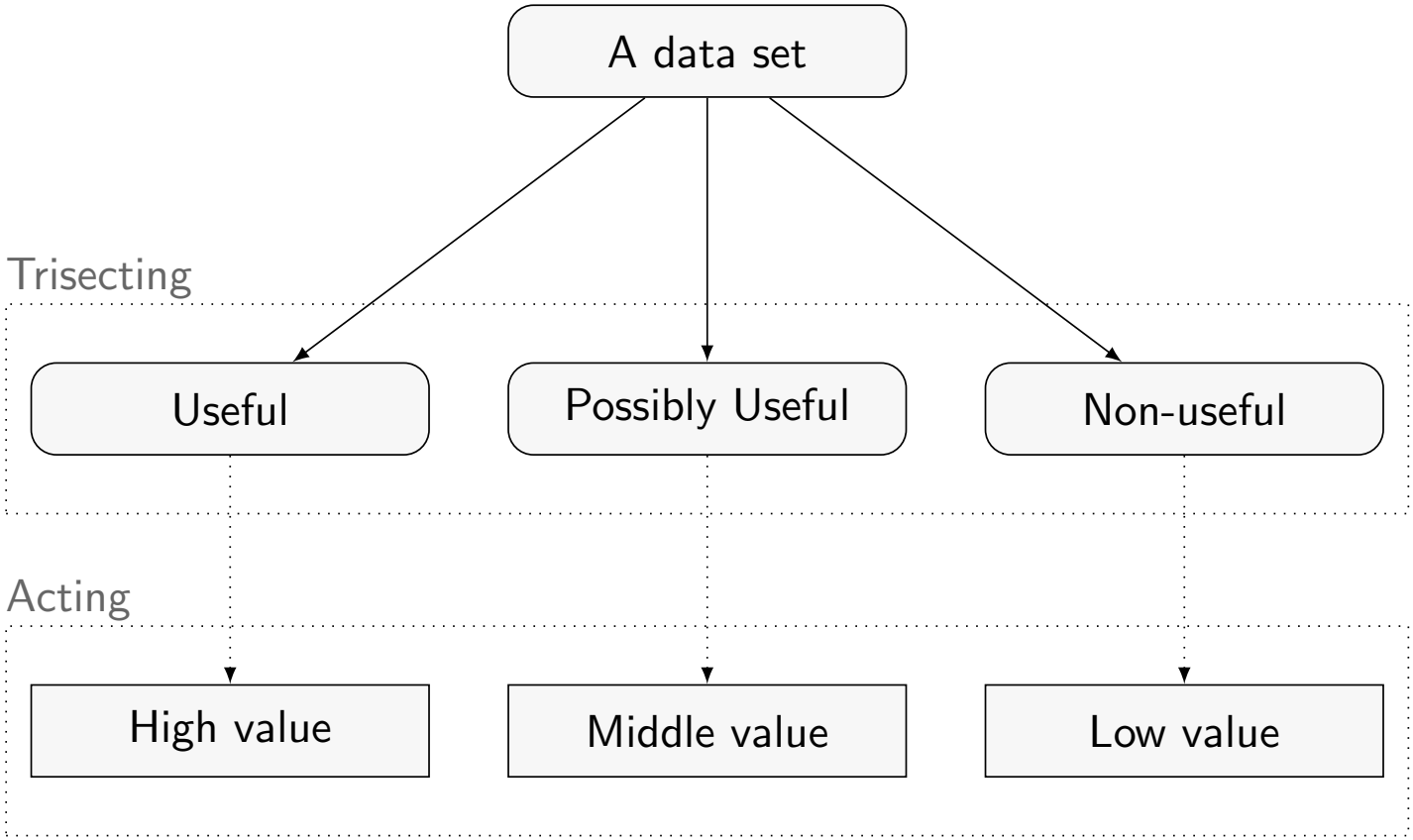
Three-way Data Selection

Pareto Principle or 20/80 Laws:



http://musicianguide.cn/8020-rule-in-music-social-media-marketing?utm_source=weixin&utm_medium=mgarticle&utm_campaign=socialmedia

Three-Way Data Selection



Three-way Data Selection

Pareto Principle or 20/80 Laws:

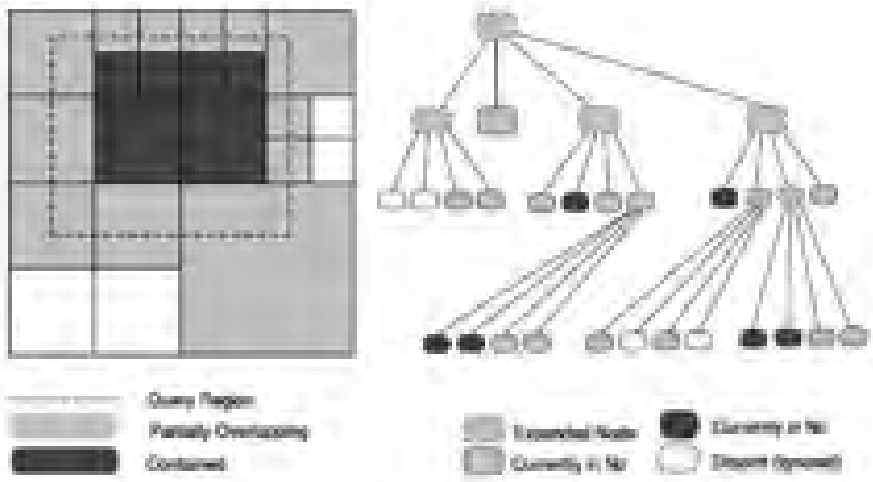


Figure 4: Query region is approximated by progressive subdivision of the MRA-tree

Lazaridis, I. and Mehrotra, S. Progressive approximate aggregate queries with a multi-resolution tree structure, 2001 ACM SIGMOD, pp. 401-412, 2001.

Sequential three-way granular data analytics

- Describe data at multiple levels of granularity.
- Design models for processing data at different level of granulaity.
- Make decisions at each levels from some object and delay decision-making for other objects.
- For deferred decisions:
  - use more complex model, and
  - use more detailed description of data,

Special Models of Three-Way Decisions

- Rough Sets
- Interval Sets
- Three-way Approximations of Fuzzy Sets
- Shadowed Sets
- Three-way Bayesian Confirmation
- Orthopairs (pairs of disjoint sets, set pairs)

Examples of 3WD research

- Theories and Models of 3WD
- Cluster Analysis
- Classification
- Email Spam Filtering
- Government Decision-Making
- Decision-Support Systems
- Sentiment Analysis
- Abnormality Detection

Three Chinese Books

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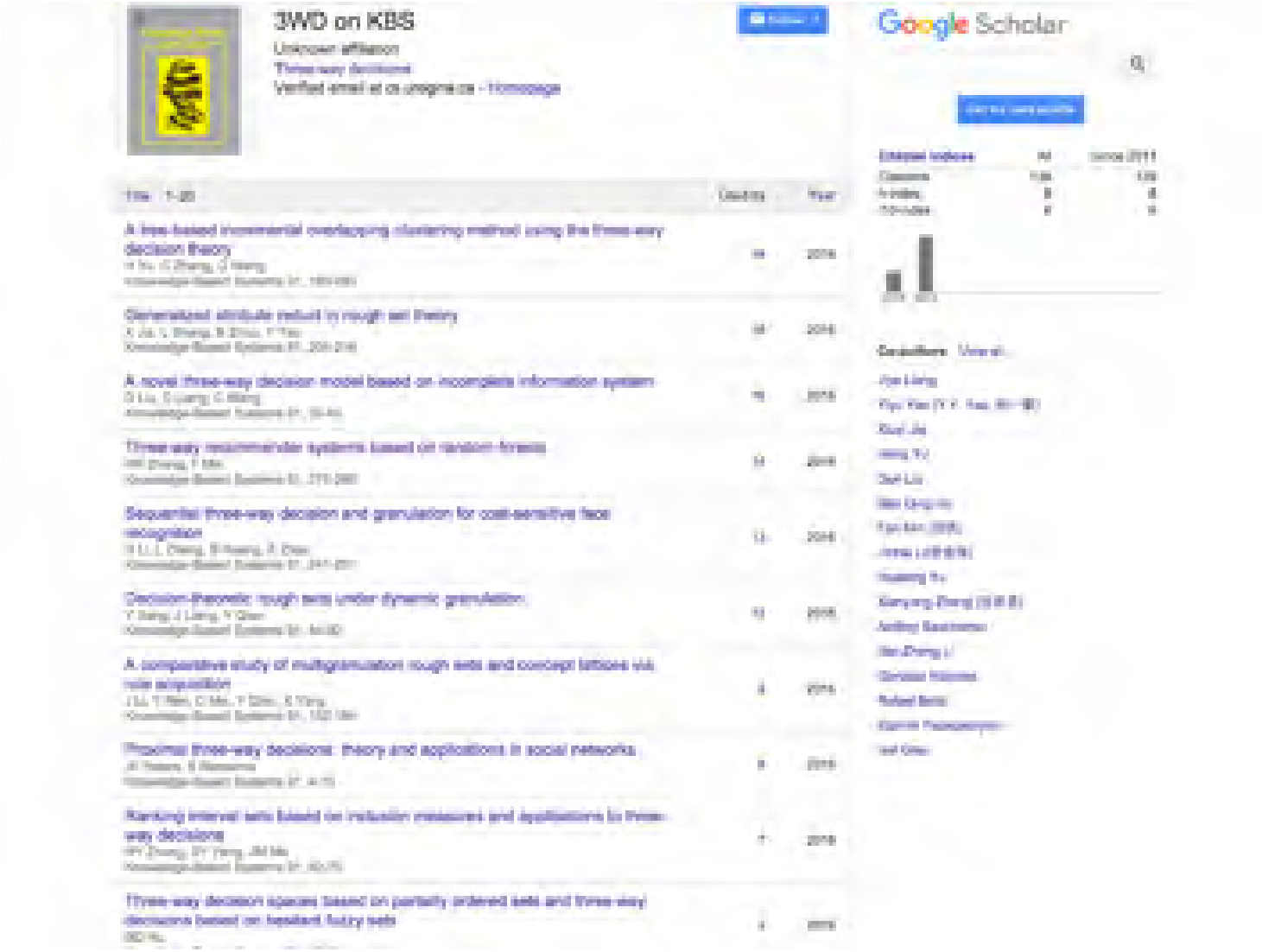
- Jia, X.Y., Shang, L., Zhou, X.Z., Liang, J.Y., Miao, D.Q., Wang, G.Y., Li, T.R., Zhang, Y.P. (Eds.), Theory of Three-way Decisions and Application, Nanjing University Press, Nanjing, China, 2012.

  贾修一，商琳，周献中，梁吉业，苗夺谦，王国胤，李天瑞，张燕平，三支决策理论与应用，南京：南京大学出版社，2012.

Special Issue

- Special issue on "three-way decisions and granular computing" in Knowledge-based System (Guest editors: Hamido Fujita, Tianrui Li, Yiyu Yao)
- Homepage in Google Scholar: `https://scholar.google.ca/citations?hl=en&user=btM_CLsAAAAJ&view_op=list_works`

Special Issue



2015 IJCRS International Rough Set Society Fellow Talks

- Tianrui Li, Chuan Luo, Hongmei Chen, Junbo Zhang: PICKT: A Solution for Big Data Analysis. RSKT 2015, pp. 15-25.
- Jiye Liang: Decision-Oriented Rough Set Methods. RSKT 2015, pp. 3-12.
- Ning Zhong, Jiajin Huang: Granular Structures Induced by Interval Sets and Rough Sets. RSKT 2015, pp. 49-60.
- Yiyu Yao: Rough Sets and Three-Way Decisions. RSKT 2015, pp. 62-73.

2016大数据决策高峰论坛

- 李天瑞, 面向大数据的动态知识发现与三支决策.
- 苗夺谦, 不确定性与三支决策.
- 于 洪, 三支决策聚类.
- Yiyu Yao: Three-Way Decisions and Cognitive Computing.

Theories and Models of 3WD

- B.Q. Hu, Three-way decision spaces based on partially ordered sets and three-way decisions based on hesitant fuzzy sets, Knowledge-Based Systems 91, 16-31, 2016.
- D. Liu, D. Liang, C. Wang, A novel three-way decision model based on incomplete information system, Knowledge-Based Systems 91, 32-45, 2016.
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- H. Dou, X.B. Yang, X. Song, H. Yu, W.Z. Wu, J. Yang, Decision-theoretic rough set: a multicost strategy, Knowledge-Based Systems 91, 71-83, 2016.
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Three-Way Cluster Analysis

- H. Yu, C. Zhang, G.Y. Wang, A tree-based incremental overlapping clustering method using the three-way decision theory, Knowledge-Based Systems 91, 189-203, 2016.
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Three-Way Classification

- H.X. Li, L. Zhang, B. Huang, X.Z. Zhou, Sequential three-way decision and granulation for cost-sensitive face recognition, Knowledge-Based Systems 91, 241-251, 2016.
- D.C. Liang, W. Pedrycz, D. Liu, P. Hu, Three-way decisions based on decision-theoretic rough sets under linguistic assessment with the aid of group decision making, Applied Soft Computing 29, 256-269, 2015.
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- S.J. Liao, Q.X., Zhu, F. Min, Cost-sensitive attribute reduction in decision-theoretic Rough set models, Mathematical Problems in Engineering 1-14, 2014

Email Spam Filtering

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Government Decision-Making

- D. Liu, T.R. Li, D.C. Liang, Three-way government decision analysis with decision-theoretic rough sets. International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems 20 (2012) 119-132.
- D. Liu, Yao, Y.Y., T.R. Li, Three-way investment decisions with decision-theoretic rough sets International Journal of Computational Intelligence Systems 4 (2011) 66-74.

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- J. Chen, Y.P. Zhang, S. Zhao, Multi-granular mining for boundary regions in three-way decision theory, Knowledge-Based Systems 91, 287-292, 2016.
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Abnormality Detection

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- Y.L. Liu, L. Pan, X.Y. Jia, Three-way decision based overlapping community detection, RSKT 2013, 279-290, 2013.

Three-way Approximations of Fuzzy Sets

- X.F. Deng, Y.Y. Yao, Decision-theoretic three-way approximations of fuzzy sets, Information Sciences 279 (2014) 702-715.

Concluding Remarks

- Three-Way Decisions are everywhere.
- Three-Way Decisions offer a new paradigm of decision making.
- There still does not exist a general theory of 3WD.
- There are both opportunities and challenges:
  - More need to be done.
  - More can be done.
  - A question that remains is "HOW?"

How?

Welcome the participant of the bright minds that are presented here today.

Current Research on Three-Way Decisions

http://www2.cs.uregina.ca/~twd/

Thank you! 谢谢!

For more information, see

http://www.cs.uregina.ca/~yyao